The Effect of Poor Scattering on the Degrees-of-Freedom in Interfering Multiple-Access Channels

Jangho Yoon†, Won-Yong Shin‡, and Hwang Soo Lee‡

†Electrical Engineering, KAIST, Daejeon 305-701, Republic of Korea
‡Computer Science and Engineering, KAIST, Daejeon 305-701, Republic of Korea
Email: yunjh@kaist.ac.kr; wyshin@dankook.ac.kr; hwanglee@kaist.ac.kr

Abstract—Opportunistic interference alignment (OIA) is known to achieve the optimal degrees-of-freedom (DoF) in the interfering multiple-access channel (IMAC) with independent and identically distributed (i.i.d.) Rayleigh fading, provided that a certain user scaling condition is satisfied. We analyze the performance of OIA in a poor scattering K-cell single-input multiple-output IMAC, where there exist finite paths between the transmitter and receiver sides. Under the feasible model, we derive a new fundamental user scaling law, required to achieve a target DoF, which generalizes the existing achievability result shown for the i.i.d. Rayleigh fading case. Our main result indicates that $KS$ DoF is achievable if the number of per-cell mobile stations (MSs) scales at least as $SNR^{(K-1) \min(L,S)}$, where $L$ denotes the number of paths and $S$ denotes the number of simultaneously transmitting MSs per cell. To verify our achievability result for finite system parameters, computer simulations are performed along with comparison to the i.i.d. Rayleigh channel case. The amount of leakage of interference is numerically evaluated and is shown to be consistent with our theoretical result. The achievable sum-rates are also evaluated.

I. INTRODUCTION

There has been a great deal of research to characterize the asymptotic capacity of interference channels using the simple notion of degrees-of-freedom (DoF), also known as multiplexing gain. Recently, interference alignment (IA) was proposed by fundamentally solving the interference problem. It was shown in [1] that the IA scheme can achieve the optimal DoF, equal to $K/2$, in the $K$-user interference channel with time-varying channel coefficients. Following up this success for IA, the underlying idea of [1] has been widely applied to various wireless network environments: multiple-input multiple-output (MIMO) interference networks [2]–[4], X networks [5], [6], and cellular networks [7]–[9].

In this work, we focus on the interfering multiple-access channel (IMAC), i.e., the multi-cell uplink network. Besides the IA scheme in [7], termed subspace IA, that achieves the optimal DoF for the IMAC model, the concept of opportunistic IA (OIA) [10], [11] was more recently introduced for the single-input multiple-output (SIMO) IMAC with time-invariant channel coefficients. The OIA scheme intelligently incorporates user scheduling into the classical IA framework in signal vector space. More specifically, the OIA scheme opportunistically selects certain mobile stations (MSs) in each cell in the sense that inter-cell interference is sufficiently small or closely aligned at a pre-defined interference space. It was shown in [11] that one can asymptotically achieve the optimal DoF if the number of MSs in a cell is beyond a certain value, i.e., if a certain user scaling condition is guaranteed. The original work in [11] was extended to the MIMO IMAC model by employing various types of pre-processing methods in [12], [13]. We note that in prior work [11]–[13], the optimal DoF and the required user scaling condition were characterized under the following assumption: the channel is independent and identically distributed (i.i.d.) Rayleigh fading (i.e., the rich scattering fading channel was assumed). However, such an assumption is hardly realistic.

In this paper, we consider the poor scattering channel with $L$ paths between the transmitter and receiver sides. Differently from [14] where the diversity of the impulse response of the channel is exploited, when the OIA framework based on the multiuser diversity is used, we show a general achievability result under the assumed poor scattering SIMO IMAC model. Then, we analyze the performance of OIA in the poor scattering $K$-cell SIMO IMAC, where each cell consists of one base station (BS) with $M$ antennas and $N$ MSs having a single antenna each. To be specific, under the model, we derive a new fundamental user scaling law, required to achieve a target DoF, as a function of $L$, which completely generalizes the achievability result shown for the i.i.d. Rayleigh fading case (i.e. $L \to \infty$) [16]. When the existing OIA protocol is applied to our system model, we characterize a generalized version of leakage of interference (LIF), qualifying the amount of induced interference to other-cell BSs, and its probability distribution, which is not straightforward. Performance is then analyzed in terms of DoF. As our main result, it is shown that $KS$ DoF is achievable if $N$ scales at least as $SNR^{(K-1) \min(L,S)}$, where $S$ denotes the number of simultaneously transmitting users per cell and $SNR$ denotes the received signal-to-noise ratio (SNR). To verify our achievability result for finite $N$ and SNR regimes, computer simulations are also performed along with comparison to the rich scattering i.i.d. Rayleigh channel case. The average amount of LIF is numerically evaluated and is shown to be consistent with the theoretically obtained LIF decaying rate, which contains the user scaling condition. The average achievable sum-rates are also evaluated according to various system parameters.

II. SYSTEM AND CHANNEL MODELS

Let us consider the IMAC model to describe realistic cellular uplink networks. Suppose that there are $K$-cells, each
of which has $N$ users equipped with a single antenna and is covered by one base station (BS). Each BS is assumed to have $M$ receive antennas and be interested only in the traffic demands of users in its cell.

The term $h_{g,s}^k \in \mathbb{C}^{M \times 1}$ denotes the uplink channel vector between the $s$th user in the $g$th cell and the $k$th BS. As illustrated in Fig. 1, we assume that the channel vector of each link between a user and BS consists of finite $L$ paths, each of which has a Rayleigh channel gain and random phase array. Then, each channel vector can be expressed as [15]:

$$ h_{g,s}^k = \sqrt{\frac{1}{L}} \sum_{l=1}^{L} \psi_{g,s}^k(l) \left[ e^{j\theta_{g,s}^k(l)(1)} \ldots e^{j\theta_{g,s}^k(M)} \right]^T $$

$$ = \sqrt{\frac{1}{L}} \left[ e_{g,s}^{k,1} \ldots e_{g,s}^{k,L} \right] \left[ \begin{array}{c} \psi_{g,s}^k(1) \\ \vdots \\ \psi_{g,s}^k(L) \end{array} \right], $$

where $\psi_{g,s}^k(l) \in \mathbb{C}$ denotes the channel gain of the $l$th path, which is assumed to have an independent complex Gaussian distribution $\mathcal{CN}(0,1)$ over $l$, and $e_{g,s}^{k,l} = \left[ e^{j\theta_{g,s}^k(l)(1)} \ldots e^{j\theta_{g,s}^k(M)} \right]^T \in \mathbb{C}^{M \times 1}$ is the phase array of the $l$th path, each of which is a unit complex number and has a random phase $\theta_{g,s}^k(l)$ uniform over $[0, 2\pi]$. As an extreme case, when $L \rightarrow \infty$, the channel in (1) is known to be equivalent to the i.i.d Rayleigh fading SIMO channel, i.e., the i.i.d. rich scattering SIMO channel [16].

Let $\Phi_g = \{ \phi_g(1), \ldots, \phi_g(S) \}$ denote the set of simultaneously active MSs who are given the opportunity to transmit their symbols in cell $g$, where $\phi_g(s) \in \{ 1, \ldots, N \}$ is an active MS in cell $g$ and $\phi_g(a) \neq \phi_g(b)$ for $a \neq b$. The parameter $S \in \{ 1, \ldots, M \}$ indicates the number of active MSs per cell and is assumed to be the same for all cells. When $S$ active MSs in each cell transmit their symbols simultaneously, the received signal vector $y_g \in \mathbb{C}^{M \times 1}$ at the $g$th BS is given by

$$ y_g = \sum_{s \in \Phi_g} h_{g,s}^g x_{g,s} + \sum_{k=1}^{K} \sum_{s \in \Phi_k} h_{g,s}^k x_{k,s} + z_g, $$

where $x_{g,s}$ is the transmit symbol of user $s$ in the $g$th cell and $z_g \in \mathbb{C}^{M \times 1}$ denotes the i.i.d. and circularly symmetric complex additive white Gaussian noise (AWGN) vector whose elements follow $\mathcal{CN}(0, N_0)$. We assume that each MS has an average transmit power constraint $\mathbb{E}[|x_{g,s}|^2] \leq P$, where $P > 0$ is a constant. The received SNR at each BS is then expressed as a function of $P$ and $N_0$.

### III. Opportunistic Interference Alignment

In this section, we briefly describe the OIA protocol [11], which intelligently combines the user scheduling and the classical IA framework. The overall procedure of the OIA scheme uses the channel reciprocity of time-division duplexing (TDD) systems. It is then possible for the MS to obtain all the cross-channel vectors, by utilizing pilot signals sent from other-cell BSs.

Each BS broadcasts its receive subspace matrix $W_g = \left[ w_{g,1}, \ldots, w_{g,S} \right]$, where $w_{g,s} \in \mathbb{C}^{M \times 1}$ is an orthonormal basis vector of $W_g$. Each MS then computes the total amount of generating interference, termed LIF [4], [10], [11]. This LIF metric is expected to affect the receive subspaces of other cells, and is given by

$$ \text{LIF}_{g,s} = \sum_{k=1}^{K} ||W_k^H h_{g,s}^k||^2. $$

Here, an arbitrarily small LIF indicates that the MS’s generating interference vectors are closely aligned at some specific subspaces, which are orthogonal to the other cell’s pre-defined receive subspaces. Through the LIF metrics from the belonging MSs, each BS selects a set of $S$ MSs who feed back the LIF values up to the $S$th smallest one in (2).

The OIA protocol basically consists of the following five steps.

- **Step 1:** Each BS randomly chooses and broadcasts a $S$-dimensional receive subspace.
- **Step 2:** Each MS computes its LIF shown in (2), and sends it to its home-cell BS.
- **Step 3:** Each BS selects the $S$ belonging MSs yielding the LIF values up to the $S$th smallest one.
- **Step 4:** The selected MSs transmit their data symbols to their home-cell BSs.
- **Step 5:** Each BS decodes the received symbols transmitted by the belonging MSs.

Each BS decodes the MSs’ symbols using the ZF matrix $G_g = \left[ g_{g,1}, \ldots, g_{g,S} \right] \in \mathbb{C}^{M \times M}$ based on the desired links and predefined receive subspace, while treating all the interference from other cells as noise, where $g_{g,s} \in \mathbb{C}^{M \times 1}$ is the $s$th ZF column vector at the $g$th BS, which perfectly removes the intra-cell interference. Then, the decoded symbols at each BS are given by

$$ \left[ \hat{x}_{g,\phi_g(1)}, \ldots, \hat{x}_{g,\phi_g(S)} \right]^T = G_g^H y_g, $$

where

$$ G_g^H = \left( W_g^H \left[ h_{g,\phi_g(1)}, \ldots, h_{g,\phi_g(S)} \right] \right)^{-1} W_g^H, $$

$\phi_g(s) \in \{ 1, \ldots, N \}$ is an active MS in cell $g$, and $\phi_g(a) \neq \phi_g(b)$ for $a \neq b$.

### IV. Achievability Result

In this section, we show our achievability results in the poor scattering IMAC model, which include the LIF decaying rate and the achievable DoF along with the user scaling law. The achievability results are also compared with the conventional i.i.d. Rayleigh channel case.

Prior to presenting our achievability results, we first characterize the cumulative density function (CDF) of the LIF under...
the poor scattering IMAC model in the following lemma. This result is used to analyze the DoF and the required user scaling law.

**Lemma 1:** In the poor scattering SIMO IMAC model with \( L \) paths, the CDF of the LIF, \( F_{\text{LIF}}(\epsilon) \), is lower-bounded by \( F_{\text{LIF}}(\epsilon) \geq C_1 \epsilon^\beta \) for \( 0 < \epsilon \leq 2MS \), where \( C_1 \) is given by

\[
C_1 = \frac{e^{-1} (1/(2MS))^{\beta}}{\Gamma(\beta + 1)}.
\]

Here, \( \beta = (K - 1) \min(L, S) \) and \( \Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \) is the Gamma function.

**Proof:** Refer to [17].

Based on Lemma 1, we show our main result, which is the characterization of the LIF decaying rate and the achievable DoF under the poor scattering IMAC model.

**A. LIF Decaying Rate**

Let \( LIF_{\text{LIF}}^{g_{\min, S}} \) denote the maximum value (i.e., the \( S \)th smallest value) among the LIFs that \( S \) selected users in the \( g \)th cell compute, which is given by

\[
LIF_{\text{LIF}}^{g_{\min, S}} = \max_{s \in \Phi_g} LIF_{g,s}.
\]

where \( \Phi_g \) is the set of simultaneously active MSs that are given the opportunity to transmit their symbols in the \( g \)th cell. Since the performance of the OIA scheme is limited mainly by such a selected MS that generates the maximum amount of interference to other-cell BSs, it is certainly worth analyzing an asymptotic behavior of \( LIF_{\text{LIF}}^{g_{\min, S}} \) with respect to \( N \).

Now, we establish the following theorem, which shows a lower bound on the LIF decaying rate with respect to \( N \).

**Theorem 1:** Under the poor scattering SIMO IMAC model, the average decaying rate of LIF is lower-bounded by

\[
E \left[ \frac{1}{LIF_{\text{LIF}}^{g_{\min, S}}} \right] \geq \Theta \left( N^{\frac{1}{\beta}} \right),
\]

where \( \beta = (K - 1) \min(L, S) \).

**Proof:** Refer to [17].

From Theorem 1, the following observation is made from system parameters such as \( L \) and \( S \).

**Remark 1:** Theorem 1 indicates that, for \( L < S \), the LIF decaying rate under the poor scattering model is much faster than the rich scattering i.i.d. Rayleigh channel case, i.e., \( L \to \infty \). This is because the probability that the amount of generating interference in the receive subspaces of the other-cell BSs is below a certain level increases as the number of paths, \( L \), is reduced. Thus, for given \( N \), the average LIF under the poor scattering SIMO IMAC model can be reduced significantly with decreasing \( L \).

---

1We use the following notation: i) \( f(x) = O(g(x)) \) means that there exist constants \( C \) and \( c \) such that \( f(x) \leq C g(x) \) for all \( x > c \). ii) \( g(x) = o(f(x)) \) means that \( \lim_{x \to \infty} \frac{f(x)}{g(x)} = 0 \). iii) \( f(x) = \Theta(g(x)) \) if \( f(x) = O(g(x)) \) and \( g(x) = O(f(x)) \).

**B. The Analysis of DoF and User Scaling Law**

Now, we show the user scaling condition such that the OIA scheme with \( S \) simultaneously active MSs in a cell achieves \( KS \) DoF asymptotically under the poor scattering IMAC model. Specifically, using the scaling argument bridging between the number of per-cell users, \( N \), and the received SNR (refer to [11], [12] for the details), we analyze 1) the achievable DoF and 2) the minimum \( N \) required to guarantee the DoF achievability result.

The total number of DoF, denoted by \( \text{DoF}_{\text{total}} \), is defined as [1]:

\[
\text{DoF}_{\text{total}} = \sum_{g=1}^{K} \sum_{s=1}^{S} \lim_{\text{SNR} \to \infty} \frac{R_g(s)(\text{SNR})}{\log \text{SNR}}, \tag{3}
\]

where \( R_g(s)(\text{SNR}) \) denotes the transmission rate of user \( s \) in cell \( g \). Under the OIA framework, the achievable \( \text{DoF}_{\text{total}} \) is lower-bounded by

\[
\text{DoF}_{\text{total}} \geq \sum_{g=1}^{K} \sum_{s=1}^{S} \lim_{\text{SNR} \to \infty} \frac{\log(1 + \text{SNR}_{g,s})}{\log \text{SNR}}, \tag{4}
\]

where \( \text{SNR}_{g,s} \) is the set of active MSs in cell \( g \), and \( \text{SNR}_{g,s} \) denotes the received signal-to-interference-and-noise ratio (SINR) at the BS for the desired signal sent from MS \( s \) in cell \( g \), which is given by

\[
\text{SNR}_{g,s} = \frac{|H_g H_g^H |^2 + \sum_{k=1}^{S} \sum_{s \in \Phi_k \setminus \Phi_g} |H_k h_k^s|^2}{1 + \text{SNR} \sum_{k=1}^{S} \sum_{s \in \Phi_k \setminus \Phi_g} \text{LIF}_{k,s}}, \tag{5}
\]

where the inequality holds due to the fact that

\[
\text{LIF}_{k,s} = \sum_{g \neq k} \|W_g H_k h_k^s\|^2 \geq \|W_g H_k^2 h_k^s\|^2 \geq \|g_g h_g^2\|^2.
\]

Note that, even if the bounding technique in (5) provides a loose bound on the SINR, it is sufficient to guarantee our achievability result. From (4) and (5), to achieve full DoF along with the OIA framework, the term \( \text{SNR} \sum_{k=1}^{S} \sum_{s \in \Phi_k \setminus \Phi_g} \text{LIF}_{k,s} \) should be arbitrarily small or remain constant, i.e., \( \text{SNR} \sum_{k=1}^{S} \sum_{s \in \Phi_k \setminus \Phi_g} \text{LIF}_{k,s} = O(1) \). Hence, it is important to analyze the distribution of the sum of LIFs in order to quantify the achievable \( \text{DoF}_{\text{total}} \).

Now, we establish our second main result, which shows the achievable DoF and the required user scaling law for the poor scattering IMAC model using the OIA protocol.

**Theorem 2:** Suppose that the OIA scheme with \( S \) simultaneously transmitting per-cell users is used in the poor scattering SIMO IMAC model. Then, under the model, \( \text{DoF}_{\text{total}} \geq KS \) is achievable with high probability if \( N = \omega \left( \text{SNR}^\beta \right) \), where \( \beta = (K - 1) \min(L, S) \).

**Proof:** A brief sketch of the proof is provided here. Let \( P_{\text{OA}} \) denote the probability that \( KS \) DoF is achievable. Then,
a lower bound on the achievable DoF is given by
\[
\text{DoF}_{\text{total}} \geq \mathcal{P}_{\text{OIA}} K S,
\]
which indicates that DoF is achievable for a fraction of the actual transmission. Now, let us focus on characterizing \( \mathcal{P}_{\text{OIA}} \). As addressed earlier, DoF is achievable if \( \text{SNR} \sum_{k=1}^{K} \sum_{g \in \Phi_k} \text{LIF}_{k,t} \) \( = O(1) \) for all \( g \in \{1, 2, \ldots, K\} \).

It is obvious that \( \text{SNR} \sum_{k=1}^{K} \sum_{g \in \Phi_k} \text{LIF}_{k,t} \) \( = O(1) \) for all \( g \in \{1, 2, \ldots, K\} \) if there exist at least \( S \) MSs in a cell satisfying \( \text{LIF}_{k,t} \leq \text{SNR} \), where \( \epsilon_0 > 0 \) is a constant independent of SNR. For this reason, \( \mathcal{P}_{\text{OIA}} \) is lower-bounded by
\[
\mathcal{P}_{\text{OIA}} \geq \left(1 - \sum_{t=0}^{S-1} \mathcal{P}_{t,N} \left(\frac{\epsilon_0}{\text{SNR}}\right)^{\frac{1}{\beta}}\right)^K,
\]
where \( \mathcal{P}_{t,N}(\epsilon) \) indicates the probability that there exist \( t \) per-cell MSs among \( N \) MSs satisfying \( \text{LIF} \leq \epsilon \) and is given by
\[
\mathcal{P}_{t,N}(\epsilon) = \frac{N}{t} \mathcal{F}_{\text{LIF}}(\epsilon) (1 - \mathcal{F}_{\text{LIF}}(\epsilon))^{N-t}.
\]

Thus, using the result of Lemma 1 in (6), we have
\[
\mathcal{P}_{\text{OIA}} \geq \left(1 - \sum_{t=0}^{S-1} \mathcal{P}_{t,N} \left(\frac{\epsilon_0}{\text{SNR}}\right)^{\frac{1}{\beta}}\right)^K \geq \left(1 - S N^{S-1} \left(1 - C_1 \left(\frac{\epsilon_0}{\text{SNR}}\right)^{\beta} \right)^{N-S+1} \right)^K,
\]
where the inequality follows due to \( \mathcal{P}_{t,N} \left(\frac{\epsilon_0}{\text{SNR}}\right)^{\frac{1}{\beta}} \leq N^{S-1} \left(1 - \mathcal{F}_{\text{LIF}} \left(\frac{\epsilon_0}{\text{SNR}}\right)\right)^{N-S+1} \) for \( t \in \{0, 1, \ldots, S-1\} \) and \( \mathcal{F}_{\text{LIF}} \left(\frac{\epsilon_0}{\text{SNR}}\right) \geq C_1 \left(\frac{\epsilon_0}{\text{SNR}}\right)^{\beta} \). In this case, it follows that
\[
\lim_{\text{SNR} \to \infty} N^{S-1} \left(1 - C_1 \left(\frac{\epsilon_0}{\text{SNR}}\right)^{\beta} \right)^{N-S+1} = 0
\]
if \( N \) scales faster than \( \text{SNR}^{\beta} \) since, in (7), the second term decays exponentially with increasing SNR while the first term increases rather polynomially. Therefore, \( \mathcal{P}_{\text{OIA}} \) asymptotically approaches one, which means that DoF is achievable with high probability if \( N = O(1) \). This completes the proof of the theorem.

Based on our achievability result, the following interesting observations are made.

Remark 2: For the rich scattering i.i.d. Rayleigh channel case, i.e., \( L \to \infty \), the term \( \min(L, S) \) is equal to \( S \), and thus the user scaling law required to achieve a target DoF is given by \( N = \omega(\text{SNR}^{K-1}) \), which is turned out to coincide with the result of Theorem 1 in [11].

Remark 3: From Theorems 1 and 2, the following valuable insight is provided: the faster LIF decaying rate with respect to \( N \), the smaller SNR exponent in the user scaling law. Note that the argument above holds for general channel models including the i.i.d. case [11].

Remark 4: Furthermore, it is examined how to achieve the non-integer DoF when the user scaling condition in Theorem 2 is not strictly satisfied for given system/channel parameters \( K \), \( L \), and \( S \). With a slight modification, it turns out that the total non-integer DoF
\[
\text{DoF}_{\text{total}} = K S (1 - \delta)
\]
for \( 0 \leq \delta \leq 1 \) can be achieved if \( N \) scales faster than \( \text{SNR}^{\beta(1-\delta)} \) where \( \beta = (K-1)\min(L, S) \). Here, the term \( 1 - \delta \) indicates the ratio of the achievable non-integer DoF to the value where a vanishingly small amount of interference is allowed. Note that this completely generalizes our achievability result in Theorem 2.

V. NUMERICAL EVALUATION

To verify our achievability result in the poor scattering IMAC model, computer simulations are performed for finite parameters \( N \) and SNR. In our simulation, the channel vector in (1) is generated \( 10^4 \) times for each system parameter.

In this work, the average amount of LIF is first evaluated numerically according to the number of MSs in each cell, \( N \).
increasing rate, representing the slope of the curves, varies the sum-rates get improved with increasing $N$.

Let us consider the case where $K = 3$ and $M = 4$. In Fig. 2, the log-log plot of LIF versus $N$ is shown as $N$ increases for various system/channel parameters $L$ and $S$, indicating the number of paths and the number of active per-cell MSs, respectively. This numerical result reveals that the LIF tends to decrease almost linearly with $N$, but the slopes of the curves vary according to $L$ and $S$. The dotted lines are obtained from Theorem 1 (theoretical results) with proper biases, and thus only the slopes of the dotted lines are relevant. It is shown that the LIF bound in Theorem 1 is in fact accurate and tight only the slopes of the dotted lines are relevant. It is shown as $N$ increases as $L$, while the slopes of the LIF curves remain almost the same when $L \geq S$. This is because the user scaling condition in Theorems 1 and 2 contains the term $\min(L, S)$, which becomes $S$ when $L \geq S$.

As illustrated in Fig. 3, the achievable sum-rates are now evaluated according to $N$. In this simulation, it is assumed that $K = 3$, $M = 4$, and SNR = 20 dB. It is obvious that the sum-rates get improved with increasing $N$, while their increasing rate, representing the slope of the curves, varies according to $L$ and $S$. It is also seen that the higher $L$, the smaller sum-rates due to the enriched path diversity for interfering links. This result reveals that the OIA under our poor scattering channel model always outperforms that under the i.i.d. Rayleigh channel assumption.

VI. CONCLUSION

Under the OIA framework, the DoF and user scaling law analysis [11] for the rich scattering i.i.d. Rayleigh fading case has been extended for the poor scattering channel model with $L$ paths between the transmitter and receiver sides. In the $K$-cell SIMO IMAC, the LIF decaying rate with respect to $N$ was first shown to scale at least as $N^{1/\beta}$ for $\beta = (K - 1) \min(L, S)$. The OIA protocol operating with $S$ active MSs in each cell was also shown to asymptotically achieve $K S$ DoF as long as $N$ scales faster than $\text{SNR}^\alpha$, which is smaller than or equal to the rich scattering fading case. The achievability results were verified in the finite $N$ regime via numerical evaluation, where the LIF decaying rate and the achievable sum-rates were evaluated for various system parameters.

ACKNOWLEDGMENT

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2012R1A1A2004947, 2014R1A1A2054577).

REFERENCES


