Large-Scale Ultra-Wide Band Ad Hoc Networks

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Abstract—We show improved throughput scaling laws for an ultra-wide band (UWB) ad hoc network, in which \( n \) wireless nodes are randomly located. First, we consider the case where a modified hierarchical cooperation (HC) strategy is used. Then, nodes are randomly located. First, we consider the case where a modified hierarchical cooperation (HC) strategy is used. While in-depth studies of HC protocols outperforms the MH scheme for certain operating regimes, due to the power-limited characteristics. It also turns out that the HC protocol is dominant for \( 2 < \alpha < 3 \) while using the nearest multi-hop (MH) routing leads to a higher throughput for \( \alpha \geq 3 \). Second, the impact and benefits of infrastructure support are analyzed; \( m \) base stations (BSs) are regularly placed in UWB networks. This case, the derived throughput scaling depends on \( \alpha \) due to the power-limited characteristics for all operating regimes examined. Furthermore, it is shown that the total throughput scales linearly with parameter \( m \) as \( m \) is larger than a certain level. Hence, we conclude that the use of either HC or infrastructure is helpful in improving the throughput scaling of UWB networks in some conditions.

I. INTRODUCTION

In [1], Gupta and Kumar introduced and characterized sum-rate scaling in a large wireless ad hoc network. They showed that, for a network of \( n \) source-destination (S–D) pairs randomly distributed in a unit area, the total throughput scales as \( \Theta(\sqrt{n / \log n}) \) [bits/Hz].\textsuperscript{1} This throughput scaling is achieved using a multi-hop (MH) communication scheme. This was improved to \( \Theta(\sqrt{n}) \) by using percolation theory [3], [4]. MH schemes are further developed and analyzed in [5], [6]. Recent research has shown that an almost linear throughput, i.e., \( \Theta(n^{1-\epsilon}) \) for an arbitrarily small \( \epsilon > 0 \), is achievable by using a hierarchical cooperation (HC) strategy [7], [8], thereby achieving the best result we can hope for in narrow-band ad hoc networks. Besides the work in [7], [8], to improve the throughput of wireless networks up to a linear scaling, novel techniques such as networks with node mobility [9], interference alignment schemes [10], and hybrid networks consisting of both wireless and infrastructure nodes [11]–[14], or equivalently base stations (BSs), have been proposed.

All the above research activities have been based on the assumption that the networks are bandwidth-constrained, i.e., narrow-band assumption. In contrast, there exists another important class of network scenarios that uses unlimited bandwidth (spectrum) resources, where the per-node transmit power is limited. Ultra-wide band (UWB) technologies are most appropriate for short range communications as well as transmissions with very low power, and thus can be developed for ad hoc sensor networks, for which the characteristics of UWB are suitable. In [15], [16], both upper and lower bounds on the capacity scaling were derived when MH schemes are applied to a UWB ad hoc network. The gap between the two bounds was then closed based on the theory of percolation [17].

In this paper, we show improved throughput scaling laws for a UWB ad hoc network, in which \( n \) wireless nodes are assumed to be randomly sited. First, we consider the case where a modified HC protocol is used. While in-depth studies of HC protocol have been conducted in narrow-band models [7], [8], such an attempt for UWB networks has never been described in the literature. We describe a HC protocol with bursty transmission. Our achievability result is based on using one of the nearest-neighbor MH scheme via percolation highway [17] and the modified HC scheme. In a dense UWB network, the result indicates that the derived throughput scaling depends on the path-loss exponent \( \alpha \) for certain operating regimes, i.e., path-loss attenuation regimes, due to the power-limited characteristics, unlike the case of narrow-band models [1], [3]–[8]. It also turns out that the use of HC is helpful in improving the throughput scaling of our UWB network in some conditions. More specifically, it is shown that the HC protocol outperforms the MH scheme for \( 2 < \alpha < 3 \), while using the MH routing leads to higher throughput for \( \alpha \geq 3 \), resulting in a highly power-limited network. Next, define the most appropriate for short range communications as well as transmissions with very low power, and thus can be developed for ad hoc sensor networks, for which the characteristics of UWB are suitable. In [15], [16], both upper and lower bounds on the capacity scaling were derived when MH schemes are applied to a UWB ad hoc network. The gap between the two bounds was then closed based on the theory of percolation [17].

In this paper, we show improved throughput scaling laws for a UWB ad hoc network, in which \( n \) wireless nodes are assumed to be randomly sited. First, we consider the case where a modified HC protocol is used. While in-depth studies of HC protocol have been conducted in narrow-band models [7], [8], such an attempt for UWB networks has never been described in the literature. We describe a HC protocol with bursty transmission. Our achievability result is based on using one of the nearest-neighbor MH scheme via percolation highway [17] and the modified HC scheme. In a dense UWB network, the result indicates that the derived throughput scaling depends on the path-loss exponent \( \alpha \) for certain operating regimes, i.e., path-loss attenuation regimes, due to the power-limited characteristics, unlike the case of narrow-band models [1], [3]–[8]. It also turns out that the use of HC is helpful in improving the throughput scaling of our UWB network in some conditions. More specifically, it is shown that the HC protocol outperforms the MH scheme for \( 2 < \alpha < 3 \), while using the MH routing leads to higher throughput for \( \alpha \geq 3 \), resulting in a highly power-limited network. Next, we take into account infrastructure-supported UWB networks having \( m \) regularly-placed BSs. While in-depth studies of BS support have been conducted in narrow-band models [11]–[14], such an attempt for UWB networks has never been done in the literature. We use the existing routing scheme [13], composed of two variants, with and without BS help, with a slight modification. The nearest-neighbor MH via percolation highway [17] is also applied for pure ad hoc transmissions with no BS support. Our result indicates that the throughput scaling always depends on the path-loss exponent for all operating

\textsuperscript{1}We use the following notation: i) \( f(x) = O(g(x)) \) means that there exist constants \( C \) and \( c \) such that \( f(x) \leq C g(x) \) for all \( x > c \). ii) \( f(x) = o(g(x)) \) means that \( \lim_{x \to \infty} f(x)/g(x) = 0 \). iii) \( f(x) = \Omega(g(x)) \) if \( g(x) = O(f(x)) \), iv) \( f(x) = \Theta(g(x)) \) if \( f(x) = O(g(x)) \) and \( g(x) = O(f(x)) \).
regimes examined while relying on parameter \( m \). It is also shown that the total throughput increases linearly with \( m \) as \( m \) exceeds a certain level, as in the narrow-band scenario [11]–[13].

The rest of this paper is organized as follows. Section II describes the system and channel models. In Section III, a modified HC protocol is described and its achievable throughput scaling is analyzed. In Section IV, our infrastructure-supported routing protocol is described and its achievability result is analyzed in terms of throughput scaling. Finally, we summarize the paper with some concluding remark in Section V.

We refer to the full papers [18], [19] for all the proofs.

II. SYSTEM AND CHANNEL MODELS

We consider a two-dimensional ad hoc network that consists of \( n \) wireless nodes uniformly and independently distributed on a square of unit area, i.e., a dense network [1], [5], [7], [8]. We randomly pick a match of S–D pairs, so that each node is the destination of exactly one source. We refer to the full papers [18], [19] for all the proofs. Furthermore, an UWB communication model is assumed, where each link operates over a relatively large bandwidth \( W \), increasing as a function of \( n \), thus yielding a power-limited (but not bandwidth-limited) system.

The basic signal model is now described as follows. The received signal \( y_k \) at node \( k \in \{ 1, \cdots, n \} \) at a given time instance is given by

\[
y_k = \sum_{i \in I} h_{ki} x_i + n_k,
\]

where \( I \subset \{ 1, \cdots, n \} \) denotes the set of simultaneously transmitting nodes, which is a subset of \( n \) transmitters available in the network, \( x_i \in \mathbb{C} \) is the signal transmitted by the \( i \)th node, and \( n_k \) denotes the circularly symmetric complex additive white Gaussian noise with zero-mean and variance \( N_0 \). Here, the complex channel gain \( h_{ki} \in \mathbb{C} \) between two nodes \( i, k \in \{ 1, \cdots, n \} \) is given by

\[
h_{ki} = \frac{e^{j \theta_{ki}}}{r_{ki}^{\alpha/2}},
\]

where \( e^{j \theta_{ki}} \) represents the random phase uniformly distributed over \([0, 2\pi]\) and independent for different \( i, k \), and time (transmission symbol), i.e., fast fading is assumed. Here, \( r_{ki} \) is the distance between nodes \( i \) and \( k \), and \( \alpha > 2 \) denotes the path-loss exponent.\(^2\) When the desired transmitter(s) is assumed to be node \( i \), the sum of the power of the received interference and noise at node \( k \) is then given by

\[
W N_0 + \sum_{i' \neq i, i' \in I} P_i |h_{k'i'}|^2,
\]

where the term \( W N_0 \) is the power of noise falling within \( W \).

In the power-constrained scenario, our system is affected by noise (but not interference) if

\[
W \gg \sum_{i' \neq i, i' \in I} \frac{P_i}{N_0} |h_{k'i'}|^2,
\]

which will be specified later.\(^3\)

Now, let us turn to an infrastructure-supported UWB ad hoc network. Suppose that the whole area is divided into \( m \) square cells, each of which is covered by one single-antenna BS at its center (see Fig. 1). It is assumed that \( n \) nodes are located except for the area covered by BSs. For analytical convenience, let us state that parameters \( n \) and \( m \) are related according to \( m = n^\beta \) for \( \beta \in [0, 1] \). Moreover, as in [11]–[14], it is assumed that the BS-to-BS links have infinite bandwidth connections each other and that these BSs are neither sources nor destinations.

In this case, the signal model in the uplink is described as follows. The received signal \( y_k \) at BS \( k \in \{ 1, \cdots, m \} \) at a given time instance is given by

\[
y_k = \sum_{i \in I} h_{ki} x_i + n_k,
\]

where \( x_i \in \mathbb{C} \) is the signal transmitted by the \( i \)th node. Here, the complex channel gain \( h_{ki} \) between node \( i \in \{ 1, \cdots, n \} \) and BS \( k \) is given by (1) when \( r_{ki} \) is the distance between node \( i \) and BS \( k \). The received signal-to-interference-and-noise ratio (SINR) at BS \( k \) from the desired transmitter \( i \) is then given by

\[
\text{SINR} = \frac{P_i h_{ki}^2}{W N_0 + \sum_{i' \neq i, i' \in I} P_i h_{k'i'}^2}.
\]

Likewise, the complex channel in the downlink between BS \( k \in \{ 1, \cdots, m \} \) and node \( i \in \{ 1, \cdots, n \} \), and the complex channel between nodes \( i, k \in \{ 1, \cdots, n \} \) can be modeled in a similar manner.

\(^2\)In [3], [4], an absorption component \( e^{-\gamma r_{ki}} \) for \( \gamma \geq 0 \) has also been incorporated in the channel model. In this work, the term \( e^{-\gamma r_{ki}} \) is not taken into account since in dense networks, it approaches a positive constant, independent of \( n \), as \( n \to \infty \), and thus does not affect scaling laws.

\(^3\)Note that in the bandwidth-limited case [1], [3]–[8], \( W = \Theta(1) \) is assumed and thus the resulting system is affected by interference.
During the second phase, a long-range multiple-input multiple-output (MIMO) transmission between two clusters having a source and its destination is performed, one at a time.

During the last phase, each node quantizes the received observations and delivers the quantized data to the corresponding destinations in the same cluster. By collecting all quantized observations, each destination can decode packets directly via single-hop to the corresponding destinations in the same cluster.

When each node transmits data within its cluster, which is performed during the first and third phases, it is possible to apply another smaller-scaled cooperation within each cluster by dividing each cluster into smaller ones. By recursively applying this procedure, it is possible to establish the hierarchical strategy in the network. We refer to [7] for more detailed description.

Due to the power-limited characteristics, our HC scheme is used with the full transmit power, i.e., the transmit power at each node is $P$. To simply apply the analysis for networks with no power limitation to our network model, instead of original (continuous) HC schemes, we utilize a bursty transmission, as similarly in [7], which uses only a fraction $\Theta(n/W)$ of the time for actual transmission with instantaneous power $WP/n$ per node and remains silent for the rest of the time (see Fig. 2). With this scheme, the received signal power from the desired transmitter(s) and the noise have the same scaling, i.e., $\Theta(W)$, and thus the (instantaneous) received signal-to-noise ratio (SNR) is kept at $\Theta(1)$ under the UWB model, which will be shown in the next section.

2) Percolation Highway Delivery Routing: We briefly introduce how to operate the MH routing via percolation highway [17] under our UWB ad hoc network, which shows the best throughput performance among the existing MH schemes [15]–[17]. The basic procedure of the percolation highway delivery follows three steps: draining, highway, and delivery phases. Let us first explain how to construct a backbone network. We divide the area into equal square grids of edge length $c_1/(2\sqrt{n})$ for a constant $c_1 > 0$, independent of $n$. Next we divide the network area into equal horizontal rectangles of size $1 \times 1 + \frac{1}{4} \log l$, which enables to generate $\Theta(\log l)$ horizontal disjoint open paths that cross each rectangle from left to right, where $l = \sqrt{2n}/c_1$. Each of the rectangles thus has $l \times \log l$ grids in the percolation model. The area can also be divided into $m/\log m$ equal vertical rectangles to generate vertical disjoint paths from bottom to top.

(i) Draining phase: A source in each horizontal rectangle sends its packets directly via single-hop to a node on a horizontal path of the backbone network.

(ii) Highway phase: The packets are transported along the horizontal path using MH routing and then reach a vertical path.

(iii) Delivery phase: A node in the vertical path sends the packets directly via single-hop to the corresponding destination.

We refer to [17] for the detailed description. Note that the average number of simultaneously active S–D pairs is given by $\Theta(\sqrt{n})$ with high probability (whp) since there exist $\Theta(\sqrt{n})$ horizontal and vertical paths simultaneously, with all the rectangles.

B. Throughput Analysis

In this subsection, we introduce and analyze the achievable throughput scaling based on the two routing protocols HC and MH shown in Section III-A. We start from the following lemma.

**Lemma 1**: In two-dimensional dense networks where $n$ nodes are uniformly distributed, the minimum distance between any two nodes is larger than $\frac{1}{n\sqrt{\log n}}$ whp.

The proof of this lemma is presented in [7]. From Lemma 1, (1), and (2), it follows that

$$\sum_{i' \neq i, i' \in I} P|h_{ki'}|^2 = \sum_{i' \neq i, i' \in I} \frac{P}{k_{ki'}} \leq Pn^{\alpha+1}(\log n)^{\alpha/2},$$

where the inequality comes from Lemma 1. Thus, if $W = \Omega(n^{\alpha+1}(\log n)^{\alpha/2})$, then the interference is negligible with respect to the noise term, resulting in a limited received signal...
power even in dense networks.\textsuperscript{4} That is, using a higher transmit power leads to more increased SINR under the condition $W = \Omega\left(n^{\alpha+1}(\log n)^{\alpha/2}\right)$, thus yielding a better throughput performance. The following result presents the achievable rate under the nearest-neighbor MH protocol.

Lemma 2: Suppose that $W = \Omega\left(n^{\alpha+1}(\log n)^{\alpha/2}\right)$. Then, $T(n) = \Omega\left(n^{(\alpha+1)/2}\right)$ is achievable whp by using the MH routing along the highway.

The proof of this lemma is presented in [17]. Based on the two protocols, we are now ready to present the total throughput $T(n)$ in the UWB ad hoc network, which is our first main result.

Theorem 1: Suppose that $W = \Omega\left(n^{\alpha+1}(\log n)^{\alpha/2}\right)$. In a dense UWB network using our routing protocol,

\[ T(n) = \Omega\left(\max\left\{n^{(\alpha+1)/2}, n^{2-\epsilon}\right\}\right) \tag{3} \]

is achievable whp for an arbitrarily small $\epsilon > 0$.

From this result, interesting observations are obtained according to operating regimes (or equivalently path-loss attenuation regimes).

Remark 1: As illustrated in Fig. 3, it turns out that the throughput scaling in (3) depends on path-loss exponent $\alpha$ for $\alpha \geq 3$ due to the fact that our considered dense network is power-limited, but not bandwidth-limited, unlike the narrow-band case [1], [3]–[8]. It is also important to examine the best between the two schemes HC and MH in each regime. For $2 < \alpha < 3$, our HC protocol outperforms the MH routing while achieving $T(n) = \Omega(n^{2-\epsilon})$ for an arbitrarily small $\epsilon > 0$. On the other hand, for $\alpha \geq 3$, using the MH protocol provides a higher throughput (i.e., $T(n) = \Omega(n^{(\alpha+1)/2})$) because our network becomes highly power-limited. In addition, we remark that the total throughput $T(n)$ quantified over the whole bandwidth $W$ does not decrease as $\alpha$ increases, whereas throughput per unit bandwidth, measured in b/s/Hz, gets reduced with increasing $\alpha$, which is rather obvious.

\textsuperscript{4}Note that the bandwidth scaling condition $W = \Omega\left(n^{\alpha+1}(\log n)^{\alpha/2}\right)$ can be scaled down by showing a tighter upper bound on the total amount of interference based on node-indexing and layering techniques similar to those in [7], [14].

Furthermore, the derived achievable rate scaling is compared with the case of narrow-band models.

Remark 2: In narrow-band ad hoc networks of unit area, an almost linear throughput is achieved using the original HC scheme. Due to bandwidth limitation, more transmit power beyond a certain level at each node does not provide a better performance on the total throughput, which is a main feature that distinguishes narrow systems from UWB ad hoc networks.

IV. IMPROVED THROUGHPUT SCALING USING INFRASTRUCTURE

Now, we show an improved throughput scaling law for UWB networks having regularly-placed infrastructure nodes.

A. Routing Protocol

In this subsection, routing protocols with and without infrastructure support are described. Especially, we utilize the best achievable scheme [13] among the conventional strategies [11]–[13], with a slight modification, in a UWB network with single-antenna BSs.

1) Infrastructure-Supported Delivery Routing: In a dense network, the BS-based MH routing scheme is described as follows:

- Divide the network into equal square cells of area $1/m$ each having one BS at the center of each cell, and again divide each cell into smaller square cells of area $2 \log n/n$ each, where these smaller cells are called routing cells, each of which includes at least one node with high probability (whp) (refer to [1], [5] for the proof).
- For the access routing, one source in each cell transmits its packets to the corresponding BS via the nearest-neighbor MH, using one of the nodes in each adjacent routing cell. The full power $P$ is used at each node.\textsuperscript{5}
- The BS that completes decoding its packets transmits them to the BS closest to the corresponding destination by wired BS-to-BS links.
- For the exit routing, the nearest-neighbor MH routing from a BS to the corresponding destination is performed, similarly to the access routing case. The transmit power at each node and BS is $P$.

\textsuperscript{5}Meanwhile, in a narrow-band model, a transmit power of $P/n^{\alpha/2}$ at each node is sufficient to guarantee the required throughput scaling since the network is bandwidth-limited (but not power-limited).

In a dense networks, the MH protocol provides a higher throughput (i.e., $T(n) = \Omega(n^{\alpha+1}/2)$) because our network becomes highly power-limited. In addition, we remark that the total throughput $T(n)$ quantified over the whole bandwidth $W$ does not decrease as $\alpha$ increases, whereas throughput per unit bandwidth, measured in b/s/Hz, gets reduced with increasing $\alpha$, which is rather obvious.
B. Throughput Analysis

In this subsection, we analyze the achievable throughput scaling based on the two routing protocols in Section IV-A. Owing to Lemmas 1 and 2, we are ready to present the achievable total throughput $T(n)$ in the UWB network with multiple BSs, which is our second main result.

**Theorem 2:** Suppose that $W = \Omega \left(n^{\alpha+1}(\log n)^{\alpha/2}\right)$. In a dense UWB network using our routing protocol,

\[
T(n) = \begin{cases} 
\Omega \left(n^{(\alpha+1)/2}\right) & \text{if } m = o\left(\sqrt{\log n}\right)^{\alpha/2} \\
\Omega \left(\frac{m^{(\alpha+1)/2}}{(\log n)^{\alpha/2}}\right) & \text{if } m = \Omega \left(\sqrt{\log n}\right)^{\alpha/2} \\
\Omega \left(n^{1-\epsilon}\right) & \text{if } m = O \left(n^{1-\epsilon}\right)
\end{cases}
\]

(4)

is then achievable whp for all $m = n^\beta$ satisfying $\beta \in [0,1)$, where $\epsilon > 0$ is an arbitrarily small constant.\(^6\)

In Fig. 4, it turns out that the throughput scaling in (4) always depends on path-loss exponent $\alpha$ for all the operating regimes $\beta \in [0,1)$, since our considered dense network is power-limited unlike the narrow-band case [11]–[13]. It is also shown how the total throughput $T(n)$ scales with respect to the number $m$ of BSs in the network. We observe that $T(n)$ does not increase as $m$ is below a certain level, in which the infrastructure is not helpful. On the other hand, as $m$ exceeds the level, the BS-based routing is dominant, as in the narrow-band model. For example, it is examined that $T(n)$ scales linearly with $m$ in the operating regime $m = \Omega \left(\sqrt{\log n}\right)^{\alpha/2}$ and $m = O \left(n^{1-\epsilon}\right)$, corresponding to $\beta \in (1/2, 1)$.

V. Conclusion

For UWB ad hoc networks of unit area, analyses have shown that the use of either HC protocol or infrastructure is helpful in improving the total throughput scaling. The sum-rate bound $T(n)$ was derived as a function of $n$ and $\alpha$ (and $m$ for the BS-based network case). It was shown that for the operating regime $2 < \alpha < 3$, our HC protocol outperforms the MH scheme, while the impact of infrastructure support is dominant as $m$ scales faster than $\sqrt{n(\log n)^{\alpha/2}}$, i.e., $\beta > 1/2$.

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\(^6\)The condition $\beta \in [0, 1)$ is needed since otherwise the result is nonsensical.