Improved Power-Delay Trade-off in Wireless Ad Hoc Networks Using Opportunistic Routing

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Abstract—We study the benefits of opportunistic routing in wireless networks by examining how the power and delay scale as the number of source-destination (S-D) pairs increases, where S-D pairs are randomly located over the network. The scaling behavior of conventional multi-hop transmission that does not employ opportunistic routing is also examined. The results indicate that the opportunistic routing can exhibit better power-delay trade-off than the conventional routing while providing up to a logarithmic boost in the scaling law. The gain comes from the fact that the system with opportunistic routing can tolerate more interference due to increased received signal power from utilizing the multi-user diversity gains. Furthermore, we derive an upper bound on the total throughput using the cut-set theorem. It is shown that the achievable rates of the conventional and opportunistic routing schemes become close to the upper bound when the number of S-D pairs is large enough.

I. INTRODUCTION

In [1], Gupta and Kumar studied the throughput scaling in large wireless ad hoc networks. They showed that a throughput scaling of \( \Theta(1/\sqrt{n \log n}) \) [bps/Hz] per S-D pair can be obtained by using a multi-hop strategy for \( n \) randomly distributed nodes in a unit area.\(^1\) This was improved to \( \Theta(1/\sqrt{n}) \) using percolation theory [3]. A recent work [4] showed that we can actually achieve \( \Theta(n^{-\epsilon}) \) scaling for an arbitrarily small \( \epsilon > 0 \) by repeatedly using a hierarchical cooperation strategy, thus achieving the best we can hope for, i.e., the linear scaling of the total throughput. Besides the throughput, other performance measures such as power and delay have been examined [5]–[7].

An important factor in practical wireless networks is the presence of multi-path fading. The effect of fading on the scaling laws was studied in [8]–[11], where the results of [8]–[10] show achievable scaling laws do not change fundamentally if the effect of fading is averaged out, while it is examined in [11] that the presence of fading can reduce the achievable throughput up to \( \log n \). However, fading can be beneficial by utilizing the opportunistic gain provided by the randomness of fading in multi-user environments [12], [13]. There is few analytical result on the usefulness of fading in wireless networks. In [14], it was shown how fading affects the throughput through the opportunistic routing [15] when there exists a single active S-D pair in a wireless network.

In this paper, we analyze the benefit of fading by utilizing opportunistic routing with multi-hop transmissions when there are multiple randomly located S-D pairs. For comparison, we also examine the performance of multi-hop transmission without opportunistic routing. It is observed that there exists a trade-off between the total transmission power per S-D pair and the average number of hops per S-D pair, i.e., delay. It is true that the power can be reduced at the expense of increased delay for both scenarios, but better trade-off can be exhibited with opportunistic routing.

As shown in [1], [3], [8]–[10], the throughput per S-D pair scales far less than \( \Theta(1) \) when the multi-hop strategy is used if there are \( n \) S-D pairs. A natural question is how scaling behaves when not all nodes in the network are sources. We are interested in the maximum number of supportive S-D pairs to maintain a constant throughput per S-D pair. We will analyze power-delay trade-off as the number of S-D pairs increases up to the maximum.

In addition, we derive an upper bound on the total throughput using the cut-set theorem. It is shown that the achievable rate of the conventional and opportunistic routing schemes becomes close to the upper bound for the case when the number of S-D pairs is large enough, i.e., when it scales faster than \( \log n \).

The rest of this paper is organized as follows. Section II describes the system model. In Section III, our routing protocols with and without opportunistic routing are described. In Section IV, the power-delay trade-off is analyzed. Section V presents the cut-set upper bound.

II. SYSTEM MODEL

We consider a wireless network that consists of \( n \) nodes uniformly and independently distributed on a torus of unit area (dense network [4]). The edge effect is removed by the symmetry introduced by the assumption of torus. We assume there are \( M(n) \) randomly located S-D pairs among \( n \) nodes, where \( M(n) \) is less than \( n \). Each source needs to transmit data to its destination at a constant rate of \( \Theta(1) \) [bps/Hz]. We assume the channel state information (CSI) is available at all receivers but not at the transmitters.
The received signal $y_j$ at node $j$ from the set $I \subset \{1,2,\ldots,n\}$ of simultaneously transmitting nodes can be written as

$$y_j = \sum_{i \in I} h_{ij} x_i + n_j,$$

where $x_i \in C$ is the transmitted signal of node $i$, $n_j$ is the complex additive-white Gaussian noise (AWGN) with zero mean and variance $N_0$, and $1 \leq j \leq n$. The channel gain $h_{ij}$ is given by

$$h_{ij} = \frac{g_{ij}}{r_{ij}^{\gamma/2}},$$

where $g_{ij}$ is the complex fading process between nodes $i$ and $j$, which is assumed to be Rayleigh with $E[|g_{ij}|^2] = 1$ and independent of each other. Moreover, the block fading model is assumed, where $g_{ij}$ is constant during one packet transmission and changes to a new independent value for the next transmission. $r_{ij}$ and $\gamma \geq 2$ denote the distance between nodes $i$ and $j$ and the path loss exponent, respectively.

### III. Routing Protocols

In this section, we describe our routing protocols with and without opportunistic routing. We use a multi-hop strategy in both cases using the nodes other than S-D pairs as relays. The average number of hops per S-D pair is interpreted as the average delay and denoted as $D(n)$. The average power per hop is given by $P(n)/D(n)$, where $P(n)$ is the total average power used by all hops for a S-D pair. Since there is no CSI at the transmitter, it is assumed that each node transmits at a fixed rate $\Theta(1)$ and at a fixed power $P(n)/D(n)$. In the next two subsections, we describe our routing protocols with and without opportunistic routing.

#### A. Opportunistic Routing

In this case, we apply opportunistic routing [15], [16] for each hop, i.e., among multiple relaying nodes that decode successfully the transmitted packet for the current hop, the one that is closest to the destination becomes the transmitter for the next hop. Short signaling messages need to be exchanged between some candidate relaying nodes to decide who will be the transmitter for the next hop. No other cooperation between relay nodes is assumed.

We divide the whole area into $1/A_s(n)$ square cells with per-cell area equal to $A_s(n)$ as shown in Fig. 1. We assume XY routing, i.e., the route for a S-D pair consists of a horizontal and a vertical path as shown in Fig. 1. Nodes operate according to the 25-time-division-multiple-access (25-TDMA) scheme. It means that the total time is divided into 25 time slots and nodes in each cell transmit 1/25-th of the time.2

All transmitters in a cell transmit simultaneously. Fig. 2 shows an example of simultaneously transmitting cells depicted as shaded cells.

2Under our routing protocol (see Fig. 2), 25-TDMA scheme is employed to guarantee that there are no transmitter and receiver nodes near the boundary of the two adjacent cells, and to avoid a hard grouping problem, which is to be discussed in this section.

Our routing protocol consists of two transmission modes, Modes 1 and 2, where Mode 2 is used for the last two hops to the destination3 and Mode 1 is used for all other hops.

**Mode 1:** We use an example shown in Fig. 3 to describe this mode. Transmitting nodes in Cell A transmit a packet simultaneously, where one of those can be either a source $S_1$ or a relay node $R_2$. A relay node that is two (Cell B) or three (Cell C) cells apart from the transmitter horizontally or vertically is chosen to transmit the packet in the next hop. When choosing the relay for the next hop, for example $R_1$ or $R_3$ in Fig. 3, one needs to consider nodes in Cells B or C that correctly decoded the packet. If there is none, then an outage occurs, whose occurrence will be maintained negligibly small. If there are more than one, then we choose one arbitrarily. Note that the multi-user diversity (MUD) gain will be roughly equal to the logarithm of the number of nodes in Cells B and C, which will be rigorously analyzed in the next section. We perform Mode 1 until the last two hops to the destination and then switch to Mode 2. The reason we hop two or three cells at a time is because 1) hopping to an immediate neighbor cell can create a huge interference to a receiver node near the boundary of the two adjacent cells and 2) always hopping by two cells is not good since it tessellates the cells into two

3Even if only one hop is needed between a S-D pair, we can artificially introduce additional hop so that there are at least two hops for every S-D pair.

4By hopping by one cell, the distance between a receiving node and an interfering node can be extremely small.
groups, even and odd, and a packet can never be exchanged between the two groups.

**Mode 2:** For the last two hops to the destination, Mode 2 is used. If we use Mode 1 for the last hop, we cannot get any opportunistic gain since the destination is predetermined. Hence, we use the following two-step procedure for Mode 2.

1. **Step 1:** In this step, a node in Cells D or E (R_D or R_E in Fig. 4) transmits its packet, whose signal reaches Cell F. This is similar to what happens in Mode 1 except we are seeing this from Cell F’s perspective. Assuming m nodes in Cell F, we arbitrarily partition Cell F into \( \sqrt{m} \) sub-cells of equal size, i.e., there are \( \sqrt{m} \) nodes in each sub-cell. One node with the best channel gain is opportunistically chosen among nodes that received the packet correctly in each sub-cell. Therefore, \( \sqrt{m} \) nodes are chosen in Cell F as potential relays for the packet.

2. **Step 2:** In Step 2, the final destination \( D_1 \) or \( D_2 \) in Cells G or H sends a short probing packet to see which one of the \( \sqrt{m} \) nodes has the best channel gain to the destination, which is selected as the relay for the next hop to the final destination.

**B. Non-Opportunistic Routing**

In this case, a plain multi-hop transmission is performed with a pre-determined path for each S-D pair consisting of a set of relays. Therefore, there is no opportunistic gain. We assume the shortest path routing. Due to fading, a transmission may not always be successful causing outages. To resolve this problem, a retransmission and/or rate/power adaptation can be used. In this paper, we simply assume there is no time-varying fading for non-opportunistic routing, and thus there will be no outage if we can control interference. This will give an upper bound on the performance.

**IV. POWER-DELAY TRADE-OFF**

If more power is available, then per-hop distance can be extended. Since the path exponent is \( \geq 2 \), the required power increases at least quadratically in the per-hop distance. On the other hand, however, the total power consumption of multi-hop is linear in the number of hops. Therefore, it seems advantageous to transmit to nearest neighbors if we want to minimize the total power. However, this comes at the cost of increased delay due to more hops. Our goal in this section is to analyze the power-delay trade-off with and without opportunistic routing. For this, we need to quantify the amount of MUD gain and the interference from other simultaneous transmissions.

In the following lemma, we first show a lower bound on the number of nodes in each cell available as potential relays.

**Lemma 1:** If \( A_s(n) = \omega(\log n/n) \), then each cell has at least \( A_s(n)n/\log n \) nodes with high probability (whp).

**Proof:** We refer readers to the full paper [17].

Note that this result shows when we can have a sufficient number of candidate nodes to relay packets at the next hop. The following lemma shows how much MUD gain can be obtained utilizing such nodes.

**Lemma 2:** If \( m = \omega(1) \), then the maximum of \( m \) squared channel gains \( |g_{ij}|^2 \) behaves like \( \Omega(\log m) \) whp.

**Proof:** We refer readers to the full paper [17].

This applies to both Modes 1 and 2. Although there are only \( \sqrt{m} \) candidate nodes in Mode 2 when it is \( m \) in Mode 1, it does not affect the scaling law since \( \log \sqrt{m} = \frac{1}{2} \log m \).

Now we turn our attention to quantifying the amount of interference in our schemes in the following two lemmas.

**Lemma 3:** If \( M(n)/D(n) = o(1) \), then the maximum number of S-D paths passing through a cell simultaneously is \( O(M(n)/D(n)) \) whp.

**Proof:** We refer readers to the full paper [17].

When we use opportunistic routing for each hop, a collision can happen, i.e., there exists a possibility that one of the receiver nodes is repeatedly selected by several transmitter nodes. In this case, one of transmitter nodes chooses the link whose channel gain is the second best, if there is no collision. Under such a routing protocol, it is guaranteed that we get the MUD gain of at least \( \Omega(\log(A_s(n)n)/\log n - M(n)/D(n)) \).
whp (in Mode 1) by combining Lemmas 1, 2, and 3. Using the result of Lemma 3, we upper-bound the total interference as a function of parameters $M(n)$, $P(n)$, and $D(n)$ in the following lemma.

**Lemma 4:** Suppose $M(n)/D(n) = \omega(1)$ and $\gamma > 2$. When the 25-TDMA scheme is used, the total interference power $P_I$ from simultaneously transmitting nodes is given by $O(P(n)M(n)D(n)^{\gamma - 2})$.

**Proof:** There are $8i$ interfering cells in the $i$-th layer of 25-TDMA. Note that the distance between a receiving node and an interfering node in the $i$-th layer is larger than $(5i - 4)/\sqrt{A_i(n)}$, where $i = 1, 2, \ldots$. The maximum number of simultaneous transmitters in each cell is given by $O(M(n)/D(n))$ by Lemma 3. Since the sum of $M(n)/D(n)$ squared channel gains $|g_{ij}|^2$ is $O(M(n)/D(n))$ whp due to $M(n)/D(n) = \omega(1)$, the effect of fading $|g_{ij}|^2$ is averaged out. Thus, the total interference power $P_I$ is given by

$$P_I = O\left(\sum_{i=1}^{\infty} \frac{(8i)P(n)/D(n)}{((5i - 4)/\sqrt{A_i(n)})^\gamma} + \frac{P(n)/D(n)}{A_i(n)^{\gamma/2}} M(n)\right)$$

$$= O\left(\frac{P(n)M(n)}{D(n)^2 A_i(n)^{\gamma/2}} \left(\sum_{i=1}^{\infty} \frac{8}{(5i - 4)^{\gamma - 1} + 1}\right)\right).$$

(3)

Since $\sum_{i=1}^{\infty} 8/(5i - 4)^{\gamma - 1}$ is bounded by a constant for $\gamma > 2$, we get

$$P_I = O\left(\frac{P(n)M(n)}{D(n)^2 A_i(n)^{\gamma/2}}\right) = O(P(n)M(n)D(n)^{\gamma - 2}),$$

(4)

which completes the proof.

Note that $P_I$ depends on the path loss exponent $\gamma$. Now we are ready to present a lower bound on the signal-to-interference-and-noise ratio (SINR) seen by any receiver.

**Theorem 1:** If $P_I = O(1)$ (i.e., $P(n)M(n)D(n)^{\gamma - 2} = \Theta(1)$), $M(n)/D(n) = \omega(1)$, $M(n) = o(n/(D(n) \log n))$, $M(n) = o(\sqrt{n}/\log n)$, and $\gamma > 2$, then the SINR is given by

$$\Omega\left(\frac{P(n)D(n)^{\gamma - 1} \log \left(\frac{n}{D(n)^2 \log n}\right)}{n}\right).$$

(5)

**Proof:** First we focus on the SINR in Mode 1. Let $P_I$ denote the received signal power from the desired transmitter. Since $P_I$ can be boosted by at least $\log(A_i(n)n)/\log n - M(n)/D(n)$) due to the MUD gain and the distance between a node in Cell A and a node in Cells B or C in Fig. 3 is less than $4\sqrt{A_i(n)}$, we have

$$P_I = \Omega\left(\frac{P(n)/D(n)}{(4\sqrt{A_i(n)})^\gamma} \log \left(\frac{A_i(n)n}{\log n} - \frac{M(n)}{D(n)}\right)\right)$$

$$= \Omega\left(\frac{P(n)D(n)^{\gamma - 1} \log \left(\frac{n}{D(n) \log n} - M(n)\right)}{D(n)^2 \log n}\right),$$

(6)

where the third equality comes from $M(n) = o(n/(D(n) \log n))$. Therefore from (1), we get

$$\text{SINR} = \frac{P_I}{N_0 + P_I},$$

(7)

which can be rewritten as (5) using Lemma 4 and (6). In Mode 2, it is straightforward to get the same result as in Mode 1 using $M(n) = o(\sqrt{n}/\log n)$ instead of $M(n) = o(n/(D(n) \log n))$ in (6).

When the throughput $T(n)$ per S-D pair is fixed at $\Theta(1)$, the scaling laws for $P(n)$ and $D(n)$ in terms of $M(n)$ can be derived by using Lemma 4 and Theorem 1 as in the following theorem.

**Theorem 2:** Assume $P_I = O(1)$ (i.e., $P(n)M(n)D(n)^{\gamma - 2} = \Theta(1)$), $M(n)/D(n) = \omega(1)$, $M(n) = o(n/(D(n) \log n))$, and $\gamma > 2$. For $T(n) = \Theta(1)$, $M(n) = o(\sqrt{n}/\log n)$, and $M(n) = \Omega(\log n)$, the power $P(n)$ and the delay $D(n)$ satisfy

$$M(n) = \Theta\left(\frac{(P(n)M(n))^{1/\gamma}}{\log n}\log \left(\frac{n(P(n)M(n))^{1/\gamma}}{(\log n)^\gamma}\right)\right).$$

(8)
and
\[ M(n) = \Theta \left( D(n) \log \left( \frac{n}{D(n)^2 \log n} \right) \right), \tag{9} \]
respectively.

**Proof:** We refer readers to the full paper [17].

Note that \( P(n) \) is monotonically decreasing with respect to \( M(n) \) while \( D(n) \) scales nearly linearly.

Finally, we compare the scaling of the opportunistic routing protocol with that of no opportunistic routing. The power-delay scaling is derived using the following lemma.

**Lemma 5:** Assume \( P_l = O(1) \) (i.e., \( P(n)M(n)D(n)^\gamma = \Theta(1) \)) and \( \gamma > 2 \). When there is no fading in our model (no opportunistic routing), i.e., \( g_{ij} = 1 \) for all \( i \) and \( j \), we get
\[ P(n) = \Theta(M(n)^{-\gamma+1}) \tag{10} \]
and
\[ D(n) = \Theta(M(n)) \tag{11} \]
under conditions that \( T(n) = \Theta(1) \), \( M(n) = o(\sqrt{n/\log n}) \), and \( M(n) = \Omega(\log n) \).

**Proof:** We refer readers to the full paper [17].

Figs. 5 and 6 show how \( P(n) \) and \( D(n) \) scale with respect to \( M(n) \). \( R_o \) and \( R_{no} \) represent the scaling curves with and without opportunistic routing, respectively. Note that the assumptions \( M(n) = o\left(D(n)^2 \log n \right) \) and \( M(n)/D(n) = \omega(1) \) in Theorem 2 always hold for \( M(n) = o(\sqrt{n/\log n}) \) and \( M(n) = \Omega(\log n) \). We observe that \( P(n) \) decreases while \( D(n) \) increases as we have more S-D pairs for both schemes. This is because we are trying to operate both schemes such that \( T(n) = \Theta(1) \), which implies that the received signal power \( P_t \) and the interference \( P_I \) needs to be \( \Omega(1) \) and \( O(1) \), respectively. This is because if \( P_I \) is not \( O(1) \), then we can scale down all powers proportionally such that \( P_I = O(1) \), since to maintain \( T(n) = \Theta(1) \), \( P_I \) must be \( \Theta(P_t) \) if \( P_t = \Omega(1) \) and having such higher power for both the signal and the interference is unnecessary. Therefore, more hops are needed to maintain the interference level at \( O(1) \) as \( M(n) \) increases. We plot the power versus the delay as shown in Fig. 7, where it is straightforward to get the power-delay trade-off curve from (8), (9), (10), and (11). It can be clearly seen that opportunistic routing \( (R_o) \) exhibits a much better power-delay trade-off than the non-opportunistic scheme \( (R_{no}) \). This gain comes from the fact that the received signal power is increased for the opportunistic routing due to the MUD gain, which allows having more interference.

### V. Cut-set Upper Bound

Our scheme by construction achieves the total throughput of \( T(n) = \Theta(M(n)) \) as long as \( M(n) \) is between \( \log n \) and \( \sqrt{n/\log n} \). In this section, we consider the cut-set upper bound [18] on the total throughput to see how close we are to the information-theoretic upper bound. We basically use the single-input multiple-output (SIMO) cut-set by separating a transmitter from the rest of the network [4], [19]. We start from the following lemma.

**Lemma 6:** In a two dimensional dense network, where \( n \) nodes are uniformly distributed randomly, the minimum distance between any two nodes is larger than \( \frac{1}{n^{(1+6)/2}} \) whp. Here, \( \epsilon > 0 \) is an arbitrarily small constant.

**Proof:** We refer readers to the full paper [17].

**Theorem 3:** The total throughput in the network with \( M(n) \) S-D pairs is upper-bounded by \( O(M(n) \log n) \) whp.

**Proof:** The proof essentially follows that of [4], [19] and Lemma 6.

Hence, it is possible to perform close to the upper bound up to \( \log n \). Note that the upper bound in Theorem 3 allows a full cooperation among all receiver nodes. When \( M(n) \) is large enough, i.e., \( M(n) = \omega(\log n) \), the achievable rate is close to the upper bound.

### References


