Effect of Multiple Antennas on the Transport Capacity in Large-Scale Ad Hoc Networks

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SUMMARY A one-dimensional ad hoc network with a single active source–destination pair is analyzed in terms of transport capacity, where each node uses multiple antennas. The analysis is based on using a multi-hop opportunistic routing transmission in the presence of fading. Specifically, the lower and upper bounds on the transport capacity are derived and their scaling law is analyzed as the node density, \(\lambda\), is assumed to be infinitely large. The lower and upper bounds are shown to have the same scaling \((\ln n)^{1/\alpha}\), where \(\alpha\) denotes the path-loss exponent. We also show that using multiple antennas at each node does not fundamentally change the scaling law.

key words: ad hoc network, fading, multi-hop, multiple antennas, opportunistic routing, scaling law, source–destination pair

1. Introduction

In a pioneering work [1], Gupta and Kumar introduced and studied the notion of transport capacity to account for the total rate-distance product carried by a large-scale wireless ad hoc network. They showed that the transport capacity obtainable by each node for a randomly chosen destination scales as \(1/\sqrt{n \log n}\) [b/s/Hz] when \(n\) source–destination (S–D) pairs using a multi-hop strategy are randomly distributed in a unit area. The results in [1] have provided some insight into the benefits of nearest-neighbor hopping, which results in the mitigation of interferences in the network. Multi-hop schemes were then further developed and analyzed in the literature [2]–[5], while their transport capacity per S–D pair scales far less than one. A recent result has shown that we can actually achieve an almost linear scaling of the total transport capacity by using a hierarchical cooperation strategy [6], thereby achieving the best result we can hope for. Besides the work in [6], to improve the aggregate capacity of wireless networks up to a linear scaling, novel techniques such as networks with node mobility [7], interference alignment schemes [8], and infrastructure support [9], [10] have also been studied.

An important factor that we need to consider in practical wireless networks is the presence of multipath fading. The effect of fading was studied in [2], [5], where it was shown that the fundamental scaling law does not essentially change if either all nodes are assumed to have their traffic demands (i.e., a heavily loaded situation with \(n\) S–D pairs is assumed) [5] or the effect of fading is averaged out [2], [5]. However, fading can be beneficial by utilizing the multiuser diversity gain provided by the randomness of fading under multiuser environments, e.g., opportunistic scheduling [11], opportunistic beamforming [12], and random beamforming [13] in broadcast channels. In [14], it was shown how fading improves the transport capacity using opportunistic routing when a single active S–D pair exists in an ad hoc network.

Our work is basically built upon [14] to analyze the transport capacity for the case where each node uses multiple antennas that commonly provide the gain through diversity and/or spatial degrees of freedom [15], [16]. In an information-theoretic point of view, it is important to know the fundamental limits of such a network scenario. The transport capacity for networks with multi-antenna nodes was shown in [17], [18] in a variety of multiuser link topologies, where no opportunistic routing was taken into account. Complementary to the transport capacity research, an alternative approach to characterizing the performance of ad hoc networks has been introduced by analyzing the transmission capacity [19]–[23], defined as the maximum spatial density of active transmitter–receiver links, multiplied by their rates, subject to an outage constraint. The transmission capacity was well studied for ad hoc networks where multiple antennas are used for various spatial multiplexing and diversity techniques [19]–[22]. Performance of multi-antenna links in ad hoc networks was also shown in terms of asymptotic spectral density [24]. Moreover, in [25], performance on the transmission capacity was characterized under either a geographical or medium access control (MAC)-induced node clustering assumption.

In this paper, we focus on a lightly loaded wireless ad hoc network having a single active S–D pair as in [14], which is an appropriate assumption for intermittent emergency traffic. We use an opportunistic routing protocol for each hop so as to utilize the randomness of fading. We then characterize the scaling law, which indicates how the transport capacity behaves as the node density, \(\lambda\), is assumed to tend to infinity. To be specific, we would like to answer the following two questions: 1) how much information our
network with multiple antennas can transport and ii) what is the effect of finite multiple antennas in terms of scaling law. As our main result, lower and upper bounds on the transport capacity are derived and their scaling law is analyzed asymptotically for high node density region. It is shown that when the absorption constant $\gamma$ becomes zero, the two bounds have the same scaling of the transport capacity, $(\ln \lambda)^{1/\alpha}$, where $\alpha$ denotes the path-loss exponent. In this case, we also show that the number of antennas at each node does not affect the scaling, while using multiple antennas still provides a better performance for finite node density region. To verify our analysis, numerical evaluation is performed.

The main contributions of this paper can be summarized as follows.

- The lower and upper bounds on the transport capacity are shown for multi-antenna ad hoc networks using an opportunistic routing.
- For the case where $\gamma = 0$, the transport capacity scaling is derived based on the two bounds using the sandwich theorem.
- Computer simulations show that the transport capacity increases with the number of antennas, which is rather obvious, while its scaling law remains constant.

The rest of this paper is organized as follows. Section 2 describes the system and channel models. In Sect. 3, the transport capacity and its asymptotic behavior for high node density are analyzed. Finally, Sect. 4 summarizes the paper with some concluding remarks.

Throughout this paper, $\| \cdot \|_F$ and $\text{tr}(\cdot)$ are the Frobenius norm and the trace, respectively, of a matrix, $E[\cdot]$ is the expectation, and $I_n$ is the identity matrix of size $n \times n$. Unless otherwise stated, all logarithms are assumed to be to the base 2.

2. System and Channel Models

Consider a one-dimensional line network model [3], [14] with a single S-D pair when a multi-hop strategy is used. Suppose that the destination is infinitely far away from the source and the nodes other than S-D pairs act as relays. Under the sufficiently random node distribution, each hop can be assumed to be independent. We thus focus on analyzing the transport capacity per hop.

We use opportunistic routing, which was originally introduced in [26], [27] and further developed in various network scenarios [28]–[30]. When a packet is sent by a transmitter, it may be possible that there are multiple receivers successfully decoding the packet. Among relaying nodes that successfully decode the transmitted packet for the current hop, the one that is closest to the destination becomes the transmitter for the next hop. Since the packet can travel farther at each hop using this opportunistic routing, the average number of hops will be reduced. In our work, we slightly modify this routing to apply it to our network with multiple antennas at each node in the presence of fading.

Short signaling messages need to be exchanged between some candidate relaying nodes and the corresponding transmit node in order to decide who will be the transmitter for the next hop. No other cooperation between relaying nodes is assumed. We assume that no channel state information (CSI) is available at the transmitter and thus the transmitter sends its packet at a fixed power and at a fixed rate.

Let $d_k$ denote the distance in meters between the transmitter and the $k$th receiver. The distance between a transmitter–receiver pair affects the received signal-to-noise ratio (SNR) at the corresponding receiver. Thus, when each node has the same number of antennas, $m$, the received signal vector $y_k$ at the $k$th node is given by

$$y_k = \sqrt{\frac{e^{-\gamma d_k} P}{d_k^\alpha m}} H_k x + n_k,$$

where $x \in \mathbb{C}^{m \times 1}$ is the transmitted signal vector with the input covariance matrix $E[xx^H] = I_m$ and $n_k \in \mathbb{C}^{m \times 1}$ is the circularly symmetric additive-white Gaussian noise vector at the $k$th node with zero mean and covariance matrix $N_0 I_n$. Here, $N_0$ is the noise spectral density. $P$, $\gamma \geq 0$, and $\alpha > 0$ denote the average transmission power per antenna, the absorption constant, and the path-loss exponent, respectively. $H_k \in \mathbb{C}^{m \times m}$ denotes the channel matrix whose element $h_{ij}^k$ represents the complex channel gain between the $i$th transmit and the $j$th receive antennas, which is assumed to be Rayleigh with $E[|h_{ij}^k|^2] = 1$ and independent for different $i, j$, and $k$. Moreover, we assume the block fading model, where $h_{ij}^k$ is constant during one packet transmission and changes independently to a new value for the next transmission. The above propagation model will be be accurate when $d_k$ is very small, but it would be asymptotically accurate under our network model. This is because as the node density $\lambda$ is infinitely large, parameter $d_k$ for the $k$th node closest to the destination also tends to infinity.

3. Transport Capacity

In this section, we shall first elaborate on the connection with transmission capacity. Then, we show the effective node density. As our main result, we derive the transport capacity and its asymptotic behavior for high node density region of our network. Some notations and definitions basically follow those of [14].

3.1 Relationship to Transmission Capacity

In this subsection, we briefly discuss the relationship between transport and transmission capacities (refer to [31] for more details).

First, let us address a common shortcoming of the transport capacity approach. While it is not possible to compute the exact transport capacity in terms of system parameters, all the results have been stated in the form of scaling laws that quantify how the total capacity grows with the
number of nodes in the network [1]–[7], [9], [10]. On the other hand, the transmission capacity framework allows for a detailed study of the critical constant term.

Next, let us focus on two important aspects that the transmission capacity framework [19]–[23] does not capture. The first is that since only simultaneous single-hop transmissions have been considered, important issues such as desired hop length, number of hops, multi-hop routes, and end-to-end delay cannot be explained. On the other hand, by enabling to route traffic from source to destination via multi-hop relaying, transport capacity measures the end-to-end sum throughput of an ad hoc network multiplied by the end-to-end distance. The second aspect is the use of only uncoordinated transmissions, which fails to schedule simultaneous transmissions with the objective of controlling interference levels, thus causing outages. Within the transport capacity framework, idealized centralized scheduling is used, where it can eliminate outages by carefully determining the optimal set of transmitters in each time slot.

Hence, transport and transmission capacities are complementary metrics. Transport capacity gives order optimal throughput optimized over all network and routing techniques, where transmission capacity gives detailed performance for the lower layers.$^1$

3.2 Bounds on the Effective Node Density

Since the channel from the transmitter to each receiving node is independent and identically distributed (i.i.d.), we omit the subscript $k$ for notational convenience. For multiple antenna systems using the input covariance matrix $I_m$ at the transmitter with no CSI, the channel capacity $C$ of each link is given by [16]

$$C = \log \det \left( I_m + \frac{e^{-\gamma s}}{\chi^m} H H^\dagger \text{SNR} \right)$$

$$= \sum_{i=1}^{m} \log \left( 1 + \frac{e^{-\gamma s}}{\chi^m} \text{SNR} \Lambda_i \right),$$

where $\text{SNR} = P/N_0W$ and $\Lambda_i$s are the nonzero eigenvalues of the channel matrix $HH^\dagger$. Here, $W$ is the bandwidth.

For the node density $\lambda$ that represents the number of nodes per meter, let us define the effective node density $\rho(x)$ at distance $x$ from the transmitters that include only the nodes that satisfy $C \geq R$, i.e.,

$$\rho(x) = \lambda \Pr \left[ \sum_{i=1}^{m} \log \left( 1 + \frac{e^{-\gamma s}}{\chi^m} \text{SNR} \Lambda_i \right) \geq R \right]. \quad (1)$$

Now we would like to consider the single antenna scenario, i.e., $m = 1$. In this case, we obtain the same result as that in [14], which is given by

$$\rho(x) = \lambda \exp \left[ -e^{-\gamma x} \left( \frac{\chi}{L} \right)^\alpha \right],$$

where $L$ is defined as

$$\log \left( 1 + \frac{e^{-\gamma L}}{L^\alpha} \text{SNR} \right) = R.$$

However, for the case where multiple antennas are used at each node, i.e., $m \geq 2$, it is not tractable to obtain the closed-form expression for the effective node density $\rho(x)$.

3.3 Transport Capacity and Its Scaling Law

In this subsection, we show how the transport capacity $C(R)$ can be computed asymptotically. To be specific, we analyze the scaling law by showing how $C(R)$ behaves as the node density $\lambda$ tends to infinity.

In our work, for a given transmission rate $R$, let us define the transport capacity $C(R)$ in bits-meters/s/Hz as

$$C(R) = E \left[ \sup_{\kappa} \max \{0, R\kappa d_k\} \right],$$

where $R_k = R$ if $C_k \geq R$ and $R_k = 0$ otherwise. Here, $C_k$ is an instantaneous channel capacity [b/s/Hz] of the link between the transmitter and the $k$th receiver. The expectation is taken over all instances of node locations and over all fading instances. Note that $\sup_x \max \{0, R\kappa d_k\} = 0$ if there is no successful reception, which means that the transport capacity would be zero due to the failed transmission, causing outages. Now, we define the transport capacity $C^*$ as

$$C^* = \sup_{R>0} C(R),$$

which is the supremum of $C(R)$ over all $R > 0$.

**Lemma 1.** For a given transmission rate $R$, the transport capacity $C(R)$ is given by

$$C(R) = R \int_0^{\infty} e^{-\gamma x} \rho(\kappa x) s p(s) ds. \quad (2)$$

The proof of this lemma is presented in [14]. Let us first introduce lower and upper bounds on the transport capacity, which are used to derive the transport capacity scaling by applying the sandwich theorem [33]. We start from defining $L_t$ and $L_u$ as

$$\log \left( 1 + \frac{e^{-\gamma L_t} \text{SNR}}{L_t^\alpha} \right) = R$$

$$m \log \left( 1 + \frac{e^{-\gamma L_u} \text{SNR}}{L_u^\alpha} \right) = R, \quad (4)$$

respectively.

**Lemma 2.** For one-dimensional networks, the transport capacity $C(R)$ is lower- and upper-bounded by

$$C(R) = R L \Phi_1(\lambda, \gamma, \alpha, m)$$

$^1$To find a middle ground between the transport and transmission capacity approaches, a new related metric, termed random access transport capacity, has recently been introduced in [32], where it computes the end-to-end throughput of multi-hop wireless networks while providing exact expressions for the pre-constants of the throughput scaling.
and
\[ C_u(R) = RL_u \Phi_1(\lambda_u, \gamma_u, \alpha, m), \tag{6} \]
respectively, where \( \lambda_1 = \lambda L_0, \lambda_u = \lambda L_u, \gamma_1 = \gamma L_0, \gamma_u = \gamma L_u, \) and
\[
\Phi_1(\lambda, \gamma, \alpha, m) = \int_0^\infty \frac{\lambda}{(m^2 - 1)!} \left( m^2 e^{-\gamma(1-\omega)}u^\alpha \right) du \exp \left[-\int_0^\infty \frac{\lambda}{(m^2 - 1)!} \left( m^2 e^{-\gamma(1-\omega)}u^\alpha \right) du \right]. \tag{7}\]

Here, \( \Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt \) is the incomplete gamma function [34].

**Proof.** From the fact that the transport capacity \( C(R) \) increases monotonically with the effective node density \( \rho(x) \), we shall show lower and upper bounds on \( \rho(x) \) so as to characterize two bounds on \( C(R) \). First, let us focus on deriving the lower bound \( C_u(R) \). From (1), it follows that
\[
\rho(x) = \lambda \Pr \left[ \sum_{i=1}^m \log \left( 1 + \frac{e^{-\gamma x}}{x^\alpha m} \right) \log \left( 1 + \frac{e^{-\gamma x}}{x^\alpha m} \right) \right] \geq R \]
\[
= \lambda \Pr \left[ \sum_{i=1}^m \frac{e^{-\gamma x}}{x^\alpha m} \right] \geq R \]
\[
= \lambda \Pr \left[ X^2_{2m^2} \geq \frac{e^{-\gamma x} m^2}{SNR} (2^R - 1) \right],
\]
where \( X^2_{2m^2} \) follows the chi-square distribution with \( 2m^2 \) degrees of freedom [23]. Here, the inequality comes from the concave characteristics of the logarithmic function. Using [23] and (3), \( \rho(x) \) is lower-bounded by
\[
\rho(x) \geq \lambda e^{-t} \sum_{i=1}^{m^2} \left[ \int_{-\infty}^{\infty} e^{-\gamma x} m^2 e^{-\gamma(1-\omega)}u^\alpha \right] \frac{t^{|\alpha|}}{|\alpha|!} \frac{\lambda}{(m^2 - 1)!} \left( m^2 e^{-\gamma(1-\omega)} \frac{x}{L_\omega} \right)^{|\alpha|} \tag{8}\]
Using (8) in (2), we obtain the following lower bound on the transport capacity:
\[
C_u(R) = R \int_0^\infty \frac{\lambda}{(m^2 - 1)!} \left( m^2 e^{-\gamma(1-\omega)} \frac{\lambda}{L_\omega} \right)^{|\alpha|} \exp \left[-\int_0^\infty \frac{\lambda}{(m^2 - 1)!} \left( m^2 e^{-\gamma(1-\omega)} \frac{\lambda}{L_\omega} \right)^{|\alpha|} \right] du \tag{9}\]
which is finally given by (5). Now let us turn to computing the upper bound \( C_u(R) \). By Jensen’s inequality, the upper bound on \( \rho(x) \) is given by
\[
\rho(x) \leq \lambda \Pr \left[ m \log \left( 1 + \frac{e^{-\gamma x}}{x^\alpha m^2} \right) \sum_{i=1}^m \frac{\Lambda_i}{R} \right] \geq R \]
\[
= \lambda \Pr \left[ m \log \left( 1 + \frac{e^{-\gamma x}}{x^\alpha m^2} \right) \sum_{i=1}^m \frac{\Lambda_i}{R} \right] \geq R \]
\[
= \lambda \Pr \left[ X^2_{2m^2} \geq \frac{e^{-\gamma x} m^2}{SNR} (2^R - 1) \right],
\]
\[
= \lambda \Pr \left[ X^2_{2m^2} \geq \frac{e^{-\gamma x} m^2}{SNR} (2^R - 1) \right],
\]
\[
= \lambda \Pr \left[ \frac{X^2_{2m^2}}{2^R - 1} \geq \frac{e^{-\gamma x} m^2}{SNR} \right],
\]
\[
= \lambda \Pr \left[ \frac{X^2_{2m^2}}{2^R - 1} \geq \frac{e^{-\gamma x} m^2}{SNR} \right],
\]
\[
= \frac{\lambda}{(m^2 - 1)!} \left( m^2 e^{-\gamma(1-\omega)} \frac{x}{L_\omega} \right)^{|\alpha|} \tag{9}\]
where the second and third equalities come from (4) and [23], respectively. Similarly as in the lower bound case, using (9) in (2) leads to
\[
C_u(R) = RL_u \int_0^\infty \frac{\lambda}{(m^2 - 1)!} \left( m^2 e^{-\gamma(1-\omega)} \frac{x}{L_\omega} \right)^{|\alpha|} \exp \left[-\int_0^\infty \frac{\lambda}{(m^2 - 1)!} \left( m^2 e^{-\gamma(1-\omega)} \frac{x}{L_\omega} \right)^{|\alpha|} \right] du \tag{9}\]
which is equal to (6). This completes the proof. \( \square \)

**Example 1.** Numerical evaluation is performed for 2 and 4 antenna systems with \( \alpha = 2, \gamma = 0, \) and \( SNR = 1. \) As illustrated in Fig. 1, the transport capacity \( C(R) \) is expressed as a function of \( R \) with various \( \lambda \)'s. It is seen that as the number of antennas, \( m, \) increases, \( C(R) \) gets improved. This means that we can transmit data at a higher rate under a fixed hop distance or increase a hop distance under a fixed rate. It is also examined that the optimal \( R \) maximizing \( C(R) \) increases as \( \lambda \) increases in all cases.

Based on the result of Lemma 2, for the case \( \gamma = 0, \) we would like to show the scaling law as follows.

**Theorem 1.** Consider the opportunistic routing. Then, the transport capacity scales as \( C^* = O((\ln \Lambda)^{1/2}) \) as \( \Lambda \) tends to infinity and \( \gamma = 0 \).

The proof of this theorem is presented in Appendix. It is now discussed how the transport capacity behaves for the case where \( \gamma > 0. \) Compared with the case \( \gamma = 0, \) since the received signal power gets reduced significantly, per-hop distance is decreased under a fixed transmission rate or data

\( ^1 \)In [23], the distribution of signal-to-interference-and-noise ratio was characterized via the Laplace transform of the interference, but it is shown to be identical to (8) if there is no interference.

\( ^{11} \) \( f(x) = O(g(x)) \) means that positive constants \( N \) and \( n \) exist such that \( f(x) \leq Ng(x) \) for all \( x > n. \)
The transport capacity $C(R)$ with respect to $R$, where $\lambda = 2^k$ for $k = 0, 1, \cdots, 8$ representing the curves from bottom to top, respectively.

Fig. 2 The transport capacity $C^*$ with respect to $\lambda$, where $m = 1 (\bigcirc)$, 2 (□), and 4 (×).

is transmitted at a lower date under a fixed hop distance. Performance on the transport capacity for finite node density region is thus degraded accordingly, which is rather obvious. However, it remains open how the presence of the absorption constant $\gamma > 0$ affects the transport capacity in terms of scaling law, which is not straightforward. From Theorem 1, we also note that the scaling $C^*$ of the transport capacity remains constant regardless of the number of antennas at each node.

Example 2. Numerical evaluation is performed to confirm the result in Theorem 1 for 1, 2 and, 4 antenna systems with $\alpha = 2$, $\gamma = 0$, and SNR = 1. Figure 2 shows the transport capacity $C^*$ versus the node density $\lambda$. It is seen that $C^*$ is unbounded as $\lambda$ increases and its asymptotes approach $(\ln \lambda)^{1/2}$.

The following example shows that although using multiple antennas does not provide a better performance in terms of scaling law, it is still beneficial for finite node density region.

Example 3. Consider the multiple antenna system with various node densities, where $\alpha = 2$, $\gamma = 0$, and SNR = 1. As illustrated in Fig. 3, it is seen that a larger transport capacity is provided compared to that of the single antenna system even for relatively high node densities. More precisely, when $\lambda = 256$, we obtain 25 to 70% performance improvement by doubling the number of antennas.

4. Conclusion

The transport capacity [1],[14] and its scaling law expression for ad hoc networks with a single active S–D pair using the opportunistic routing protocol have been extended to multiple antenna systems. Specifically, we derived the lower and upper bounds on the transport capacity $C(R)$. The lower and upper bounds on $C(R)$ were shown to have the same tendency with respect to the node density $\lambda$ asymptotically. In consequence, the scaling law of $C(R)$ was shown to be $O((\ln \lambda)^{1/\alpha})$ for every $\alpha > 0$ when the network topology is one-dimensional. It was also shown that for the finite number of multiple antennas at each node, we obtain the exactly same scaling of $C(R)$ as that in the single antenna case. Further work in this area includes extending the analysis to a two-dimensional network topology.

References


Appendix: Proof of Theorem 1

When $\gamma = 0$, the lower bound $\Phi_{1}(H, 0, \alpha, m)$ is given by

$$\Phi_{1}(H, 0, \alpha, m) = \int_{0}^{\infty} \frac{v_{1}}{(m^{2} - 1)!} \Gamma(m^{2}, m^{2}v^{\alpha}) \cdot \exp \left[ -\int_{0}^{\infty} \frac{\lambda^{1}}{(m^{2} - 1)!} \Gamma\left( m^{2}, m^{2}u^{\alpha} \right) du \right] dv$$

$$= \int_{0}^{\infty} \frac{\lambda^{1}}{m}\left( \frac{m^{2} - 1}{\lambda^{1}} \right) \cdot \frac{m^{2} - 1}{\lambda^{1}} \sum_{r=0}^{m^{2}-1} \frac{1}{r!} \left( \frac{1}{\alpha}, m^{2}v^{\alpha} \right) \cdot \exp \left( -m^{2}v^{\alpha} \right)$$

$$= \int_{0}^{\infty} \frac{\lambda^{1}}{m}\left( \frac{m^{2} - 1}{\lambda^{1}} \right) \cdot \frac{m^{2} - 1}{\lambda^{1}} \sum_{r=0}^{m^{2}-1} \frac{1}{r!} \left( \frac{1}{\alpha}, m^{2}v^{\alpha} \right) \cdot \exp \left( -m^{2}v^{\alpha} \right)$$

where the second equality holds since

$$\int_{0}^{\infty} \frac{\lambda^{1}}{(m^{2} - 1)!} \Gamma\left( m^{2}, m^{2}u^{\alpha} \right) du = \frac{\lambda^{1}}{m}\left( \frac{m^{2} - 1}{\lambda^{1}} \right) \cdot \frac{m^{2} - 1}{\lambda^{1}} \sum_{r=0}^{m^{2}-1} \frac{1}{r!} \left( \frac{1}{\alpha}, m^{2}v^{\alpha} \right).$$

It is observed that $\exp \left[ -\frac{\lambda^{1}}{m}\sum_{r=0}^{m^{2}-1} \frac{1}{r!} \left( \frac{1}{\alpha}, m^{2}v^{\alpha} \right) \right]$ exhibits a sharp transition from 0 to 1 at $v = v^{*}$ as $\lambda$ tends
yields if dropping $(\Phi)$

\[ \int_{c_i}^{\infty} \lambda^i \sum_{t=0}^{m-1} \frac{\left(m^2 v^\alpha t\right)^t}{t!} e^{-m^2 v^\alpha} \, dv, \]

which is equivalent to \( \frac{1}{e} \) from (A-1)

\[ \text{which can be rewritten as} \]

\[ \ln \lambda - m^2 v^\alpha + \left( \frac{1}{\alpha} - 1 \right) \ln \left( m^2 v^\alpha \right) + \ln \left( \sum_{t=0}^{m-1} \frac{1}{t!} (m^2 v^\alpha)^t \right) = O(1), \]

Note that 1/e is set as a certain point between 0 and 1 to give the best constant factor in the scaling law. Since \( v^\alpha \) goes to infinity as \( \lambda \to \infty \), using \( \Gamma(a,x) = e^{-x} x^{a-1} (1 + O(1/x)) \) yields

\[ \ln \lambda - m^2 v^\alpha + \left( \frac{1}{\alpha} - 2 \right) \ln \left( m^2 v^\alpha \right) = O(1) \]

using \( \sum_{t=0}^{m-1} \frac{1}{t!} (m^2 v^\alpha)^t ) = x^{m-1} \left( \frac{m^2 v^\alpha}{m^2 v^\alpha - 1} \right) + O(1/x) \), thus yielding

\[ m^2 v^\alpha - \ln \lambda = O(\ln v^\alpha). \]

Hence, we conclude that \( v^\alpha = O \left( \left( \ln \lambda \right)^{1/\alpha} \right) \) and \( C^* (R) \) scales as \( (\ln \lambda)^{1/\alpha} \), which completes the proof.

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\[ \text{To simplify notations, } \Phi_i (\lambda, 0, \alpha, m) \text{ will be written as } \Phi_i \text{ if dropping } (\lambda, 0, \alpha, m) \text{ does not cause any confusion.} \]