Hierarchical Cooperation in Ultra-Wide Band Ad Hoc Networks*

Won-Yong SHIN[3a], Nonmember and Koji ISHIBASHI[3b], Member

SUMMARY We show an improved throughput scaling law for an ultra-wide band (UWB) ad hoc network by using a modified hierarchical cooperation (HC) strategy; the $n$ wireless nodes are assumed to be randomly sited. In a dense network of unit area, our result indicates that the derived throughput scaling law depends on the path-loss exponent $\alpha$ for certain operating regimes due to the power-limited characteristics. It also turns out that the use of HC is helpful in improving the throughput scaling of our UWB network in some conditions. More specifically, assuming that the bandwidth scales faster than $n^{\alpha+1} \log n$, it is shown that the HC protocol outperforms nearest multi-hop routing for $2 < \alpha < 3$ while using nearest multi-hop routing leads to higher throughput for $\alpha \geq 3$.

key words: hierarchical cooperation (HC), multi-hop, path-loss exponent, power-limited, throughput scaling, ultra-wide band (UWB)

1. Introduction

In [1], Gupta and Kumar introduced and characterized sum-rate scaling in a large wireless ad hoc network. They showed that, for a network of $n$ source-destination (S–D) pairs randomly distributed in a unit area, the total throughput scales as $\Theta(\sqrt{n}/\log n)$ [b/s/Hz]. This throughput scaling is achieved using a multi-hop (MH) communication scheme. This was improved to $\Theta(\sqrt{n})$ by using percolation theory in [2], [3]. MH schemes are further developed and analyzed in [4], [5]. Recent research has shown that an almost linear throughput, i.e., $\Theta(n^{1-\epsilon})$ for an arbitrarily small $\epsilon > 0$, is achievable by using a hierarchical cooperation (HC) strategy [6], [7], thereby achieving the best result we can hope for in narrow-band ad hoc networks. Besides the work in [6], [7], to improve the throughput of wireless networks up to a linear scaling, novel techniques such as networks with node mobility [8], interference alignment schemes [9], and infrastructure support [10], [11] have been proposed.

All the above research activities have been based on the assumption that the networks are bandwidth-constrained, i.e., narrow-band assumption. In contrast, there exists another important class of network scenarios that uses unlimited bandwidth (spectrum) resources, where the per-node transmit power is limited. Ultra-wide band (UWB) technologies are most appropriate for short range communications as well as transmissions with very low power, and thus can be developed for ad hoc sensor networks, for which the characteristics of UWB are suitable. In [12], [13], both upper and lower bounds on the capacity scaling were derived when MH schemes are applied to a UWB ad hoc network. The gap between the two bounds was then closed based on the theory of percolation [14].

In this letter, we introduce and analyze an improved achievable throughput scaling law for UWB ad hoc networks by using a modified HC protocol; the $n$ wireless nodes are assumed to be randomly sited. While in-depth studies of HC protocol have been conducted in narrow-band models [6], [7], such an attempt for UWB networks has never been described in the literature. We first describe a HC protocol with bursty transmission. Our achievability result is based on one of the nearest-neighbor MH scheme via percolation highway [14] and the modified HC scheme. In a dense UWB network, the result indicates that the derived throughput scaling depends on the path-loss exponent $\alpha$ for certain operating regimes, i.e., path-loss attenuation regimes, due to the power-limited characteristics, unlike the case of narrow-band models [1]–[7]. It also turns out that the use of HC is helpful in improving the throughput scaling of our UWB network in some conditions. More specifically, it is shown that the HC protocol outperforms the MH scheme for $2 < \alpha < 3$, while using the MH routing leads to higher throughput for $\alpha \geq 3$, resulting in a highly power-limited network.

The rest of this paper is organized as follows. Section 2 describes the system and channel models. A modified HC protocol is described in Sect. 3 and its achievable throughput scaling is analyzed in Sect. 4. Finally, we summarize the paper with some concluding remark in Sect. 5.

Throughout this paper, $C$ denotes the field of complex numbers. Unless otherwise stated, all logarithms are assumed to be to the base 2. We also use the following asymptotic notation: i) $f(x) = O(g(x))$ means that there exist constants $C$ and $c$ such that $f(x) \leq Cg(x)$ for all $x > c$. ii) $f(x) = o(g(x))$ means that $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$. iii) $f(x) = \Omega(g(x))$ if $g(x) = O(f(x))$. iv) $f(x) = \Theta(g(x))$ if $f(x) = O(g(x))$ and $g(x) = O(f(x))$ [15].
2. System and Channel Models

We consider a two-dimensional ad hoc network that consists of \( n \) wireless nodes uniformly and independently distributed on a square of unit area, i.e., a dense network \([1, 4, 6, 7]\). We randomly pick a match of S–D pairs, so that each node is the destination of exactly one source. Suppose that each node has an average transmit power constraint \( P \) (constant) over the whole system bandwidth and transmits at a rate \( T(n)/n \) [b/s], where \( T(n) \) denotes the total throughput of the network. Furthermore, an UWB communication model is assumed, where each link operates over a relatively large bandwidth \( W \), increasing as a function of \( n \), thus yielding a power-limited (but not bandwidth-limited) system.

The signal model is now described as follows. The received signal \( y_k \) at node \( k \in \{1, \cdots, n\} \) at a given time instance is given by \( y_k = \sum_{i=1}^M h_i x_i + n_k \); where \( M \subseteq \{1, \cdots, n\} \) denotes the set of simultaneously transmitting nodes, which is a subset of \( n \) transmitters available in the network, \( x_i \in \mathbb{C} \) is the signal transmitted by the \( i \)th node, and \( n_k \) denotes the circularly symmetric complex additive white Gaussian noise with zero-mean and variance \( N_0 \). Here, the complex channel gain \( h_{ki} \in \mathbb{C} \) between two nodes \( i, k \in \{1, \cdots, n\} \) is given by

\[
h_{ki} = \frac{e^{j\theta_{ki}}}{\sqrt{r_{ki}}},
\]

where \( e^{j\theta_{ki}} \) represents the random phase uniformly distributed over \([0, 2\pi]\) and independent for different \( i, k \), and time (transmission symbol), i.e., fast fading is assumed. Here, \( r_{ki} \) is the distance between nodes \( i \) and \( k \), and \( \alpha > 2 \) denotes the path-loss exponent\(^\dagger\). When the desired transmitter(s) is assumed to be node \( i \), the sum of the power of the received interference and noise at node \( k \) is then given by

\[
W N_0 + \sum_{\ell \neq i, \ell \in I} P|h_{k\ell}|^2,
\]

where the term \( W N_0 \) is the power of noise falling within \( W \). In the power-constrained scenario, our system is affected by noise (but not interference) if \( W \gg \sum_{\ell \neq i, \ell \in I} \frac{\sum_{\ell \neq i, \ell \in I} P|h_{k\ell}|^2} {N_0} \), which will be specified later\(^\dagger\).

3. Routing Protocol

In this section, we describe a modified HC strategy based on bursty transmission, which runs the hierarchical scheme only a certain fraction of the time. For comparison, we also show the conventional nearest-neighbor MH routing scheme \([14]\) in a UWB ad hoc network.

3.1 Modified Hierarchical Cooperation

Based on the earlier studies \([1, 6, 11, 16]\) for narrow-band ad hoc networks, it follows that using the HC strategy is preferred at bandwidth-limited regimes. In UWB ad hoc networks, we introduce our modified HC scheme to identify the operating regimes (or path-loss attenuation regimes) such that HC has a better throughput performance than MH routing. HC consists of three phases as follows.

(i) Divide the network into clusters each having \( M \) nodes.
(ii) During the first phase, each source distributes its data to the other \( M - 1 \) nodes in the same cluster.
(iii) During the second phase, a long-range multiple-input multiple-output (MIMO) transmission between two clusters having a source and its destination is performed, one at a time.
(iv) During the last phase, each node quantizes the received observations and delivers the quantized data to the corresponding destinations in the same cluster. By collecting all quantized observations, each destination can decode its packet.

When each node transmits data within its cluster, which is performed during the first and third phases, it is possible to apply another smaller-scaled cooperation within each cluster by dividing each cluster into smaller ones. By recursively applying this procedure, it is possible to establish the hierarchical strategy in the network. We refer to \([6]\) for more detailed description.

Due to the power-limited characteristics, our HC scheme is used with the full transmit power, i.e., the transmit power at each node is \( P \). To simply apply the analysis for networks with no power limitation to our network model, instead of original (continuous) HC schemes, we utilize a bursty transmission, as similarly in \([6, 16]\), which uses only a fraction \( \Theta(n/W) \) of the time for actual transmission with instantaneous power \( WP/n \) per node and remains silent for the rest of the time (see Fig. 1). With this scheme, the received signal power from the desired transmitter(s) and the noise have the same scaling, i.e., \( \Theta(W) \), and thus the (instantaneous) received signal-to-noise ratio (SNR) is kept at \( \Theta(1) \) under the UWB model, which will be shown in the next section.

3.2 Percolation Highway Delivery Routing

In this subsection, we briefly introduce how to operate the

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\(^\dagger\) In \([2, 3]\), an absorption component \( e^{-\gamma x} \) for \( \gamma \geq 0 \) has also been incorporated in the channel model. In this work, the term \( e^{-\gamma x} \) is not taken into account since in dense networks, it approaches a positive constant, independent of \( x \), as \( n \to \infty \), and thus does not affect scaling laws.

\(^\dagger\) Note that in the bandwidth-limited case \([1–7]\), \( W = \Theta(1) \) is assumed and thus the resulting system is affected by interference.
MH routing via percolation highway [14] under our UWB ad hoc network, which shows the best throughput performance among the existing MH schemes [12]–[14]. The basic procedure of the percolation highway delivery follows three steps: draining, highway, and delivery phases. Let us first explain how to construct a backbone network.

We divide the network area into equal horizontal rectangles of size $1 \times \frac{1}{m} \log l$, which enables to generate $\Theta(\log l)$ horizontal disjoint open paths that cross each rectangle from left to right, where $l = \sqrt{2\pi n}/c_1$. Each of the rectangles thus has $l \times \log l$ grids in the percolation model. The area can also be divided into $m/\log m$ equal vertical rectangles to generate vertical disjoint paths from top to bottom.

(i) Draining phase: A source in each horizontal rectangle sends its packets directly via single-hop to a node on a horizontal path of the backbone network.

(ii) Highway phase: The packets are transported along the horizontal path using MH routing and then reach a vertical path.

(iii) Delivery phase: A node in the vertical path sends the packets directly via single-hop to the corresponding destination.

We refer to [14] for the detailed description. Note that the average number of simultaneously active S–D pairs is given by $\Theta(\sqrt{n})$ with high probability (whp) since there exist $\Theta(\sqrt{n})$ horizontal and vertical paths simultaneously, with all the rectangles.

4. Analysis of Throughput Scaling

In this section, we introduce and analyze the achievable throughput scaling based on the two routing protocols HC and MH shown in Sect. 3. We start from the following lemma.

**Lemma 1.** In two-dimensional dense networks where $n$ nodes are uniformly distributed, the minimum distance between any two nodes is larger than $\frac{1}{\sqrt{n} \log n}$ whp.

The proof of this lemma is presented in [6]. From Lemma 1, (1), and (2), it follows that

$$\sum_{i \neq j, i \neq k} P_{ij|k} |^2 = \sum_{i \neq j, i \neq k} P \leq Pn^{\alpha+1}(\log n)^{\alpha/2},$$

where the inequality comes from Lemma 1. Thus, if $W = \Omega(n^{\alpha+1}(\log n)^{\alpha/2})$, then the interference is negligible with respect to the noise term, resulting in a limited received signal power even in dense networks. That is, using a higher transmit power leads to more increased signal-to-interference-and-noise ratio (SINR) under the condition $W = \Omega(n^{\alpha+1}(\log n)^{\alpha/2})$, thus yielding a better throughput performance. The following result presents the achievable rate under the nearest-neighbor MH protocol.

**Lemma 2.** Suppose that $W = \Omega(n^{\alpha+1}(\log n)^{\alpha/2})$. Then, $T(n) = \Omega(n^{(\alpha+1)/2})$ is achievable whp by using the MH routing along the highway.

The proof of this lemma is presented in [14]. Based on the two protocols, we are now ready to present the total throughput $T(n)$ in the UWB ad hoc network, which is our main result.

**Theorem 1.** Suppose that $W = \Omega(n^{\alpha+1}(\log n)^{\alpha/2})$. In a dense UWB network using our routing protocol,

$$T(n) = \Omega\left(\max\{n^{(\alpha+1)/2}, n^{2-\epsilon}\}\right)$$

is achievable whp for an arbitrarily small $\epsilon > 0$.

**Proof.** We first show the throughput scaling based on the use of HC protocol. Let us focus on the narrow-band system in which HC operates with the transmit power $P/n$ [6]. In this case, the interference power received by a node from simultaneously transmitting (undesired) nodes is bounded by a constant, independent of $n$, and the received signal power from the desired transmitter(s) is given by $\Theta(1)$, thus yielding $\text{SINR} = \Theta(1)$ in all three phases (refer to [6] for the detailed analysis).

Now turn our attention to a UWB network under the bandwidth scaling assumption $W = \Omega(n^{\alpha+1}(\log n)^{\alpha/2})$. Then, for the same transmit powers, the received SINR at each node is all decreased by a factor of $W$ increasing with $n$, compared to the narrow-band network case. Since our network is power-limited, the average per-node transmit power required to run our HC scheme is not $P/n$ but the full power $P$, which leads to $\text{SINR} = \Theta(n/W)$, or equivalently $\text{SNR} = \Theta(n/W)$ from the fact that the interference scales slower than the noise term. Owing to the bursty HC scheme, the received SNR is thus kept at $\Theta(1)$ for a fraction $\Theta(n/w)$ of the time with instantaneous power $WP/n$ at each node. Hence, it follows that

$$T(n) = \Theta\left(W\left\{\frac{n}{T_{NB}(n)}\right\}\right) = \Theta(nT_{NB}(n)) = \Omega(n^{2-\epsilon})$$

whp, where $T_{NB}(n)$ [bits/Hz] denotes the throughput scaling of narrow-band networks using HC and $\epsilon > 0$ is an arbitrarily small constant.

In addition, from Lemma 2, when we consider the MH transmission through a percolation highway [14], we have $T(n) = \Omega(n^{(\alpha+1)/2})$ whp. Based on the two achievability results, it is possible to achieve a lower bound on the total throughput given by (3), which completes the proof. □

From this result, interesting observations are obtained according to operating regimes (or equivalently path-loss attenuation regimes).

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*Note that the bandwidth scaling condition $W = \Omega(n^{\alpha+1}(\log n)^{\alpha/2})$ can be scaled down by showing a tighter upper bound on the total amount of interference based on node-indexing and layering techniques similar to those in [6], [11].*
Remark 1. As illustrated in Fig. 2, it turns out that the throughput scaling in (3) depends on path-loss exponent $\alpha$ for $\alpha \geq 3$ due to the fact that our considered dense network is power-limited, but not bandwidth-limited, unlike the narrow-band case [1]–[7]. It is also important to examine the best between the two schemes HC and MH in each regime. For $2 < \alpha < 3$, our HC protocol outperforms the MH routing while achieving $T(n) = \Omega(n^{2-\epsilon})$ for an arbitrarily small $\epsilon > 0$. On the other hand, for $\alpha \geq 3$, using the MH protocol provides a higher throughput (i.e., $T(n) = \Omega(n^{\alpha/(\alpha+1)})$) because our network becomes highly power-limited. In addition, we remark that the total throughput $T(n)$ quantified over the whole bandwidth $W$ does not decrease as $\alpha$ increases, whereas throughput per unit bandwidth, measured in b/s/Hz, gets reduced with increasing $\alpha$, which is rather obvious.

Furthermore, the derived achievable rate scaling is compared with the case of narrow-band models.

Remark 2. In narrow-band ad hoc networks of unit area, an almost linear throughput is achieved using the original HC scheme. Due to bandwidth limitation, more transmit power beyond a certain level at each node does not provide a better performance on the total throughput, which is a main feature that distinguishes narrow systems from UWB ad hoc networks.

5. Conclusion

For UWB ad hoc networks of unit area, analyses have shown that use of the HC protocol is helpful in improving the total throughput scaling. The sum-rate bound $T(n)$ was derived as a function of $n$ and $\alpha$. For the operating regime $2 < \alpha < 3$, it was shown under the bandwidth scaling assumption $W = \Omega\left(n^{\alpha+1}(\log n)^{\alpha/2}\right)$ that our HC protocol outperforms the MH scheme while achieving $T(n) = \Omega(n^{2-\epsilon})$ for a sufficiently small $\epsilon > 0$. Future work in this area includes showing the order optimality of our routing protocol in UWB ad hoc networks by deriving a cut-set upper bound.

References